



Consumption and Savings Decisions: Teaching The Precautionary Motive In Intermediary Macroeconomics Courses

Most undergraduate macroeconomics courses do not address the precautionary motive, an essential factor behind savings decisions. This motive arises when future income is uncertain; consequently, we must use models in which consumers live for several periods, and their future income is treated as a random variable. To simplify the exposition, we consider a model with two periods in which future income is merely positive (employed) or null (unemployed). In this framework, we illustrate how the precautionary motive leads consumers to save more due to unemployment risk. After exposure to this material, we hope that students who have not yet studied the precautionary motive can understand it and incorporate it into their analysis of consumer behavior. In addition, teachers can use this as a guide to approach this important topic.

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1. Introduction

Undergraduate macroeconomics courses introduce the subject of consumption through the Keynesian approach in which the consumption level follows a fixed rule, where a fraction of disposable income is consumed. Consequently, savings is determined as a residual by the remaining fraction of the disposable income. Moving forward, undergraduate students learn simplified versions of the life cycle model (LCM) and the permanent income hypothesis (PIH), where consumers are aware of the intertemporal trade-off between present and future consumption, but future income is not uncertain.¹ Hence, such versions of the LCM and PIH do not answer the question of how consumers react to income uncertainty, which is an essential factor behind consumer behavior. For this reason, it is valid to present a simplified model that addresses such an issue.

To accomplish this task, we analyze the saving decisions through the conventional approach of expected utility maximization, assuming a one-good economy. Thus, consumers choose consumption and savings to maximize lifetime utility, subject to a budget constraint. We first investigate consumer behavior assuming a deterministic future income. Next, we introduce income uncertainty to identify how consumer behavior changes. As detailed in Section 2, when future income is deterministic, optimal consumption and savings depend only on current and future income levels. For comparison purposes, we define S_D as the optimal savings in this environment with deterministic future income.

Consider, then, that future income is uncertain. Intuitively, the consumer would save more to be prepared for possible negative income shocks, which is known as the precautionary motive for savings. In this vein, the optimal consumption and savings depend on future income risk instead of only its expected value. The most straightforward cause of uncertainty in future income is the chance of losing the job. Accordingly, we assume the following probability distribution for future income: $Y > 0$ with probability $1-p$, and zero with probability p . Of course, the probability of being employed is $1-p$, while the probability of losing the job is p . We define S_U as the optimal savings in this environment (with uncertainty). We know from Leland (1968) that $S_U - S_D > 0$, provided that the third derivative of the utility function is positive, as detailed in Section 3. In this case, when future income is uncertain instead of deterministic, the consumer saves more.² This additional amount of savings is called precautionary savings (Carroll & Kimball, 2008).³

It is not appropriate to limit undergraduate students to the study of consumption models in which future income is deterministic. Ignoring the uncertainty of future income precludes their understanding of essential economic phenomena, such as saving decisions.⁴ For this reason, our primary goal is to put forward a simple consumption model that captures the precautionary motive and allows us to perform exercises (simulations) useful for teaching this subject. To accomplish such a task, we adopt a treatable utility function whose third derivative is positive, namely, the *constant relative risk aversion* (CRRA). Also, we consider the most straightforward framework possible in which the consumer lives for two periods and there are

¹Modigliani and Brumberg (1954) developed the original LCM, while Friedman (1957) put forward the PIH.

²As discussed in Section 3, the distribution of future income is such that its expected value equals the deterministic income. This allows us to verify how the optimal savings reacts to the uncertainty of income rather than to changes in the level of income.

³Precautionary savings is often called *buffer stock* savings. As explained by Carroll (1992, p. 62) in “the buffer-stock model, consumers hold assets mainly so that they can shield their consumption against unpredictable fluctuations in income; unemployment expectations are therefore important because typically the most drastic fluctuations in a household’s income are those associated with spells of unemployment.”

⁴Consumer behavior can change dramatically, even when a small amount of uncertainty is introduced, if agents display prudence, that is, the third derivative of the utility function is positive (Browning & Lusardi, 1996).

two possible values for future income, one positive (employed) and another null (unemployed). To calculate the precautionary savings, we must compare the optimal savings from this model with that from a model without uncertainty. For this reason, we also analyze the case where income is deterministic. After all, the main contribution of this work is an appropriate exposition of the precautionary motive for undergraduate students since it is complementary material for teaching consumption and saving decisions in undergraduate macroeconomic courses.⁵

In addition to this introduction, this paper is organized as follows. In Section 2, we discuss different factors that affect savings, including the precautionary motive, and we briefly discuss the Keynesian formulation for the consumption function. In Section 3, we present preliminary concepts useful for understanding our two-period model with and without uncertainty, and we show that the precautionary motive emerges in the latter. Section 4 presents a road map to use this material, including an example to bring the topic to students' lives. Finally, in Section 5, we summarize the main lessons learned from the models discussed in this paper.

2. Motives for savings

The basic idea of LCM and PIH is that saving is future consumption (Romer, 2012). Once the resources are accumulated, regardless of the motive for this, future consumption is boosted. Of course, this accumulation is made by sacrificing current consumption. Therefore, there is an intertemporal trade-off between current and future consumption, which is mediated by savings decisions.

In macroeconomic courses, we study the evolution of aggregate variables; therefore, the adoption of intertemporal models is essential. In this sense, models in which consumers are aware that more consumption today means less consumption tomorrow are preferable. According to this claim, we should use models with at least two time periods. However, it is not necessary to consider various goods. Therefore, we consider two-period models in which a single consumption good that represents the aggregate consumption exists.

Before introducing the consumption models, it is instructive to review different motives for savings. Browning and Lusardi (1996) cite the eight motives for saving listed by J. Maynard Keynes (1936) and add a ninth:⁶

1. "To build up a reserve against unforeseen contingencies" (the precautionary motive);
2. "To provide for an anticipated future relationship between the income and the needs of the individual..." (the life-cycle motive);
3. "To enjoy interest and appreciation..." (the intertemporal substitution motive);
4. "To enjoy a gradually increasing expenditure..." (the improvement motive);
5. "To enjoy a sense of independence and the power to do things, though without a clear idea or definite intention of specific action" (the independence motive);
6. "To secure a masse de manoeuvre to carry out speculative or business projects" (the enterprise motive);
7. "To bequeath a fortune" (the bequest motive);

⁵As a reviewer argues this material is also appropriate for first-year master's students or students in upper-level economic courses.

⁶The designations in parentheses come from Browning and Lusardi (1996).

8. "To satisfy pure miserliness, i.e., unreasonable but insistent inhibitions against acts of expenditure as such" (the avarice motive);
9. To accumulate deposits to buy houses, cars, and other durables (the down payment motive).

In Section 3, we present two versions of a two-period consumption model. In the first version, there is no uncertainty, and the consumer uses savings to smooth the consumption path over time; this is the life-cycle motive for savings. Hence, the desire to achieve a stable consumption path throughout the lifetime determines the savings decision. The second version of the two-period model introduces uncertainty by means of the unemployment risk. To be precise, the consumer may lose his/her job in the second period. As a result, the precautionary motive emerges, stimulating consumers to save for a "rainy day". Therefore, the model takes into account two factors from the list above: the life cycle and the precautionary motives.

Before analyzing these models in Section 3, it is instructive to briefly review the Keynesian consumption function, which is given by:

$$C = C_0 + cI, \quad (1)$$

where consumption, C , is a linear function of disposable income, I , and the slope coefficient is the marginal propensity to consume, $c \in (0, 1)$. As a result, the savings level is given by:

$$S = Y - C = -C_0 + (1 - c)I, \quad (2)$$

where the coefficient $1-c$ is the marginal propensity to save. Therefore, in this model, there is no consumption nor savings decision; rather, there are only fixed behavioral rules. It is curious that Keynes' (1936) approach does not consider the motives for saving that he put forward. By adopting microfounded models instead of fixed behavioral rules, we can capture the life cycle and the precautionary motives for savings, as detailed in Section 3.

3. Precautionary Savings

Before introducing the consumption models, in Section 3.1 we present statistical tools and microeconomic concepts useful for understanding the precautionary motive. After that, we present two versions of a two-period consumption model. In Section 3.2, we present the version without any uncertainty. In Section 3.3, we add the unemployment risk. By comparing the optimal savings level in both cases, we identify the precautionary savings. In other words, we quantify the additional amount of savings due to unemployment risk.

A. Preliminary Concepts

Random variables, expected value, and variance

When future income is deterministic, the consumers know its value in advance. When there is uncertainty, the future income is a **random variable**, which is a variable that records, in numerical form, the possible outcomes from a random event. In our case, the possible outcomes are positive income (employed) or null income (unemployed). The **probability density function** describes the probabilities associated with the possible outcomes from a random variable. In our case, the probabilities are of employment and unemployment.

Following Browning and Lusardi (1996), we define the random variable associated with future income, \tilde{Y}_2 , as follows:

$$\tilde{Y}_2 = \begin{cases} \frac{Y_2}{1-p}, & \text{Prob} = 1-p, \\ 0, & \text{Prob} = p \end{cases}, \quad (3)$$

where $p \in [0,1)$ is the probability of unemployment. Thus, in the second period, the consumer will either be employed with positive income, $Y_2/(1-p)$ or unemployed (zero income). The normalization of income by $1-p$ is explained below.

Because we do not know the outcome of a random variable in advance, it makes sense to ask what its expected value is. Additionally, the variability of the possible outcomes is interesting information. These quantities are often calculated using the **expected value** and the **variance** operators. The former calculates the outcome of the random variable that will occur "on average". In our case, it is given by the multiplication of each possible outcome by the corresponding probability, i.e.,

$$E[\tilde{Y}_2] = \frac{Y_2}{1-p} \times (1-p) + 0 \times p = Y_2. \quad (4)$$

The variance of a random variable measures the dispersion of its possible expected value outcomes. In our case, the variance is given by:

$$\text{Var}(\tilde{Y}_2) = \left[\frac{Y_2}{1-p} - E[\tilde{Y}_2] \right]^2 \times (1-p) + \left[0 - E[\tilde{Y}_2] \right]^2 \times p. \quad (5)$$

Given that $E[\tilde{Y}_2] = Y_2$, the variance (5) simplifies to:

$$\text{Var}(\tilde{Y}_2) = \frac{p}{1-p} Y_2^2. \quad (6)$$

The normalization of Y_2 by $1-p$ makes the variance of the future income an increasing function of p , while the expected value of future income does not depend on p . Hence, the larger the unemployment probability, the larger the variance of future income, although the expected value of future income remains the same. This feature allows us to isolate the effect of income uncertainty on the consumer's savings decision and, as a result, we can properly quantify the precautionary motive.

Table 1 illustrates how the expected value and the variance of the future income respond to changes in the unemployment probability, assuming that $Y_2 = 1$. The expected value of future income remains constant as we keep Y_2 fixed. It is easy to see that the future income dispersion (variance) increases with the unemployment probability p .⁷ Therefore, this probability is an appropriate measure of the income uncertainty in our framework.

⁷The derivative of the variance (6) with respect to p is $[Y_2/(1-p)]^2$, being positive because $Y_2 > 0$ and $p \in [0,1)$.

Table 1 – Unemployment probability, expected value and variance of the future income uncertainty for $Y_2=1$

Unemployment probability p	Employment probability $1-p$	Expected future income Y_2	Variance of future income $\frac{p}{1-p} Y_2^2$
0.01	0.99	1	0.01
0.05	0.95	1	0.05
0.1	0.9	1	0.11
0.15	0.85	1	0.18
0.2	0.8	1	0.25

Utility Function, Risk Aversion, and Prudence

As mentioned in Section 2, we focus on the intertemporal trade-off between present and future consumption, instead of the intratemporal trade-off among different goods, and, for this reason, we consider a one-good economy.⁸ Thus, we suppose that consumer well-being depends on the instantaneous utility function, $u(C)$ where C is the consumption level. As usual, we suppose that $u(C)$ is increasing, $u'(C) > 0$ and $u''(C) < 0$, concave. Hence, the utility level is increasing in consumption at a decreasing rate.

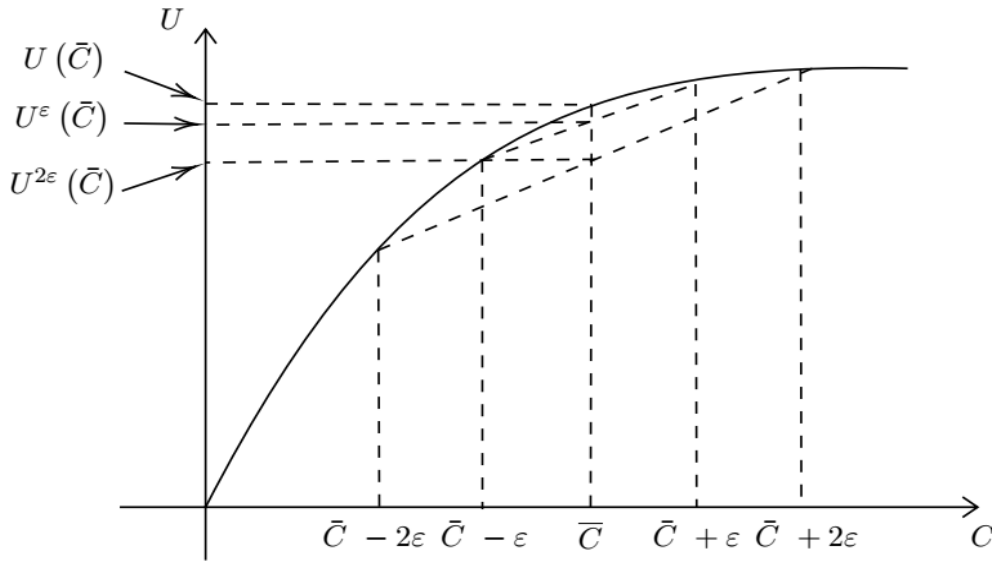
Figure 1 illustrates an increasing and concave utility function. The utility from the certain level of consumption \bar{C} is given by $U(\bar{C}) = u(\bar{C})$. Suppose now that there are two fair gambles: a 50-50 chance of consuming $\bar{C} - \varepsilon$ or $\bar{C} + \varepsilon$, and a 50-50 chance of consuming $\bar{C} - 2\varepsilon$ or $\bar{C} + 2\varepsilon$, where $\varepsilon > 0$ is a constant. Thus, the consumption is uncertain – it is not known in advance. The expected utility of gamble 1 – the utility evaluated in each possible outcome times the corresponding probability – is given by:

$$U^\varepsilon(\bar{C}) = \frac{1}{2}u(\bar{C} - \varepsilon) + \frac{1}{2}u(\bar{C} + \varepsilon), \quad (7)$$

and the expected utility of gamble 2 is given by:

$$U^{2\varepsilon}(\bar{C}) = \frac{1}{2}u(\bar{C} - 2\varepsilon) + \frac{1}{2}u(\bar{C} + 2\varepsilon). \quad (8)$$

⁸ Of course, to capture the intratemporal trade-off, a multivariate utility function should be adopted. In such a case, the economy would have more than one good.

Figure 1 – Preference over a risky consumption

As can be seen in Figure 1, the consumer prefers the assured value, instead of the gambles ($U(\bar{C}) > U^{\epsilon}(\bar{C})$ and $U(\bar{C}) > U^{2\epsilon}(\bar{C})$). Thus, the consumer prefers the certain consumption level \bar{C} , instead of any gamble in which he/she loses or gains the same amount of consumption with equal probability. An individual who refuses fair gambles is said to be **risk averse**. Furthermore, consumers prefer the small gamble to the large one ($U^{\epsilon}(\bar{C}) > U^{2\epsilon}(\bar{C})$). Intuitively, the large gamble implies greater risk, because the dispersion of the possible outcomes is larger.

The most used measures of risk aversion were introduced by Pratt (1964). They are the coefficients of absolute and relative risk aversion that are defined respectively as:

$$R_a \equiv -\frac{u''(C)}{u'(C)} \quad \text{and} \quad R_r \equiv -\frac{u''(C)C}{u'(C)}.$$

Not by chance, these coefficients are positive provided that the utility function is concave ($u''(C) < 0$), as in Figure 1.

To analyze choices under uncertainty, the researchers often adopt the CRRA utility, given by:

$$u(C) = \begin{cases} \frac{C^{1-\gamma}}{1-\gamma} & \text{if } \gamma > 0 \text{ and } \gamma \neq 1, \\ \ln C & \text{if } \gamma = 1 \end{cases} \quad (9)$$

As its name suggests, this utility has a constant relative risk aversion coefficient, being equal to the parameter γ , i.e. $R_r = \gamma$.

Therefore, when preferences are represented by the CRRA utility, the consumers dislike risk. But, how do consumers react when they face risk? To be precise, given income uncertainty, how do consumers react to the risk of losing their job? Kimball (1990) shows that

the adjustment in savings due to the uncertainty of future income depends on the sign of the third derivative of the utility function. In accordance with Leland's (1968) previous analysis, he shows that a positive third derivative (convex marginal utility) implies precautionary savings, which means that consumers save more. This happens because the expected marginal utility of savings increases as the risk increases. In our case, the larger the unemployment risk, the larger the marginal utility of savings.

Kimball (1990) also shows that adjustment in savings is proportional to what he defines as the index of absolute prudence: $P_A \equiv -u'''(C)/u''(C)$. Not by chance, this index is positive provided that the third derivative of the utility function is positive. Additionally, he defines the index of relative prudence: $P_R \equiv -u'''(C)C/u''(C)$. This index is related to the adjustment of the savings-consumption ratio due to the income uncertainty.

Notice that, the third derivative of the CRRA utility is equal to $\gamma(1+\gamma)C^{-\gamma-2}$, being positive because of $\gamma > 0$. Indeed, $P_A = (1+\gamma)C^{-1} > 0$ and $P_R = 1 + \gamma > 0$. Thus, under CRRA utility, the savings adjustment due to income uncertainty is positive because the coefficient of relative risk aversion is positive. Although the derivation of the risk aversion and the prudence coefficients goes beyond the scope of this work, the use of the CRRA utility is suitable for studying the precautionary motive. Thus, we adopt this utility.

B. Model without uncertainty

Consider a consumer living for two periods, whose well-being depends on lifetime utility, as follows:

$$U(C_1, C_2) = u(C_1) + \beta u(C_2), \quad (10)$$

where $\beta \in (0,1)$ is the intertemporal discount factor, $u(\cdot)$ is the instantaneous utility function, and C_1 and C_2 are, respectively, first- and second-period consumption levels.⁹ We assume that the instantaneous utility function is the CRRA (see equation (9)). The maximization of the welfare function (10) is subject to the following budget constraints:

$$C_1 = Y_1 - S \quad (11)$$

$$C_2 = Y_2 + RS. \quad (12)$$

The first constraint implies that first-period consumption (C_1) is determined by the difference between first-period income (Y_1) and savings (S). The second constraint implies that second-period consumption (C_2) is equal to second-period income (Y_2) plus financial wealth (RS), where $R=1+r$ is the gross rate of return on savings, with $r > 0$.

Finally, the consumer solves the following problem:

$$\begin{aligned} & \max_{C_1, C_2, S} u(C_1) + \beta u(C_2) \\ & \text{s. t. } \begin{cases} C_1 = Y_1 - S \\ C_2 = Y_2 + RS \\ C_1 \geq 0 \\ C_2 \geq 0 \end{cases} \end{aligned} \quad (13)$$

Before solving problem (13), let us analyze the marginal utility of consumption. The instantaneous utility (9) has marginal utility equal to $u'(C) = C^{-\gamma}$. As $\gamma > 0$, the marginal utility

⁹The intertemporal discount factor expresses the importance of the future relative to the present.

becomes infinitely larger as consumption trends to zero. Therefore, we know that problem (13) has an interior solution. Consequently, we can ignore the inequality constraints and substitute the equality constraints (11) and (12) in the lifetime utility (10). As a result, the consumer problem becomes:

$$\max_S u(Y_1 - S) + \beta u(Y_2 + RS). \quad (14)$$

Of course, after choosing optimal S , the optimal levels of consumption (C_1 and C_2) are determined, respectively, by budget constraints (11) and (12).

Considering the CRRA utility (9), the first-order condition of problem (14) yields the optimal savings level:¹⁰

$$S^* = \frac{(R\beta)^{1/\gamma} Y_1 - Y_2}{(R\beta)^{1/\gamma} + R}. \quad (15)$$

Although our focus is the analysis of savings, it is easy to find the solution for present and future consumption. Substituting the optimal savings (15) in the budget constraints (11) and (12), we conclude that:

$$C_1^* = \frac{RY_1 + Y_2}{(R\beta)^{1/\gamma} + R}, \quad (16)$$

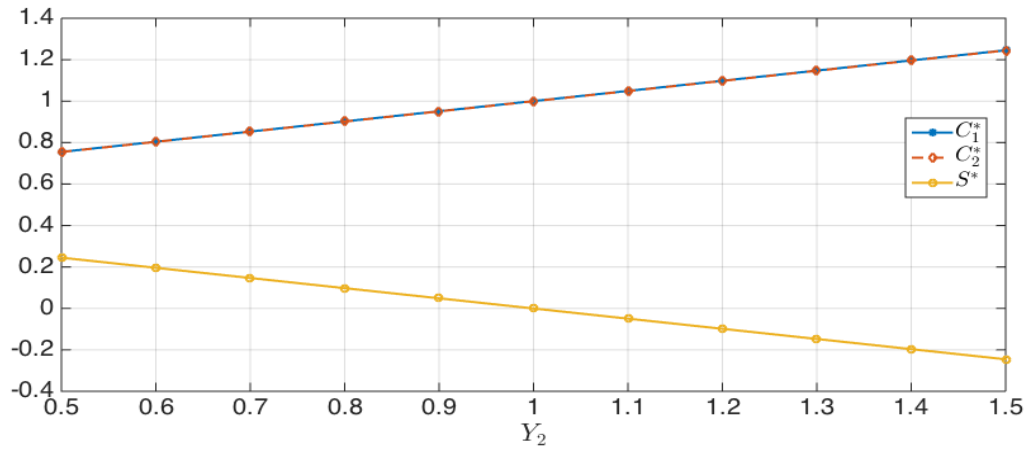
$$C_2^* = \frac{(R\beta)^{1/\gamma} (RY_1 + Y_2)}{(R\beta)^{1/\gamma} + R}. \quad (17)$$

Note that the higher the income in any period, the higher the consumption in the two periods. Savings, on the other hand, increases with current income and decreases with future income, as expected.

Figure 2 illustrates the results of this model. We set $\beta = 0.96$, $\gamma = 2.00$, $R = 1.04$ and $Y_1 = 1$.¹¹ Future income (Y_2) is given along the horizontal axis, while the vertical axis presents the optimal savings and current and future consumption. Note that Y_2 varies from 0.50 to 1.50. Since $Y_1 = 1$, we can interpret the optimal decisions as a function of the relative income Y_2/Y_1 . As expected, when the ratio Y_2/Y_1 is close to 1, the optimal savings is practically null. On the other hand, when Y_2 is lower than Y_1 , the consumer saves a fraction of Y_1 to increase future consumption. For example, when $Y_2 = 0.6$ (and $Y_1 = 1$), the consumer saves approximately 0.20. As a result, the consumption is smoothed to approximately 1 in both periods. Finally, in the opposite situation, the consumer makes a loan to smooth the consumption path. For example, when $Y_2 = 1.4$ (and $Y_1 = 1$), savings are approximately -0.20. Consequently, consumption in both periods is approximately 1.20. The use of savings to smooth the consumption path is precisely the life-cycle motive for savings.

¹⁰The necessary condition for an extremum (maximum or minimum) is for the first-order derivative to be equal to zero. To prove if an extremum is a maximum, we need to verify the second-order condition. However, since $\gamma > 0$, the CRRA function is concave, and we do not present the second-order condition for the two-period models analyzed.

¹¹As βR is approximately 1, the more important factors in equations (9), (10), and (11) are the incomes, Y_1 and Y_2 , and the gross rate of return, R .

Figure 2 – Optimal consumption and savings: effects of future income.

Therefore, this basic model captures the life-cycle motive for savings: optimal savings smooth the consumption path. The next model adds unemployment risk, and therefore, savings have a new role to protect the consumer against unemployment shock.

C. Model with uncertainty

Following Browning and Lusardi (1996), we assume that future income \tilde{Y}_2 , is described by equation (3). As a result, the variance of the future income, $\frac{p}{1-p} Y_2^2$, depends on unemployment probability, while its expected value does not (see equations (4) and (6)). These features allow us to isolate the effect of income risk on the consumer's savings decision by changing. As a result, we can properly quantify the precautionary motive.

Due to income uncertainty, the consumer's well-being is measured by the expected lifetime utility as follows:

$$E[u(C_1) + \beta u(\tilde{C}_2)], \quad (18)$$

where the notation \tilde{C}_2 indicates that future consumption is uncertain. The consumer maximizes the expected lifetime utility (18) subject to the budget constraints (11) and (12). As in Section 3.2, we use these budget constraints to substitute out current and future consumption in the objective function (18). As a result, the consumer problem simplifies to:

$$\max_S E[u(Y_1 - S) + \beta u(\tilde{Y}_2 + RS)]. \quad (19)$$

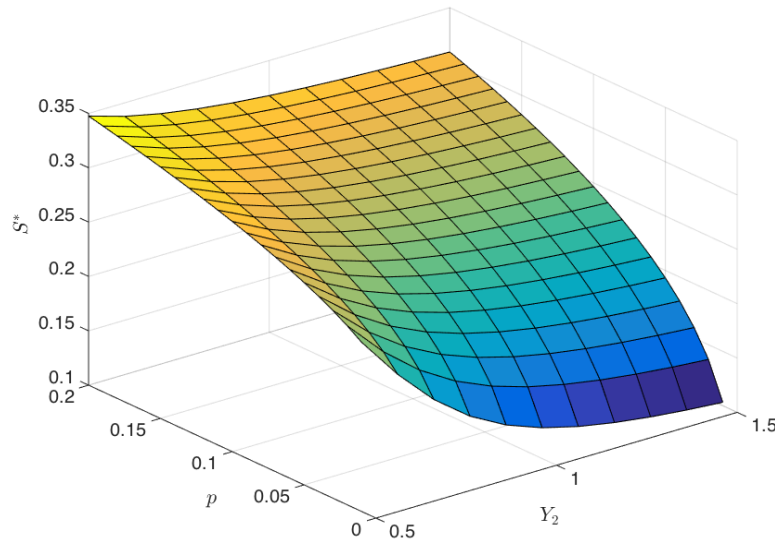
Considering the CRRA utility (9) and the future income distribution (3), the first-order condition of problem (19) yields:

$$(Y_1 - S^*)^{-\gamma} = \beta \left[(1-p) \left(\frac{Y_2}{1-p} + RS^* \right)^{-\gamma} + p(RS^*)^{-\gamma} \right] R, \quad (20)$$

where S^* is the optimal savings. The left-hand side of condition (20) is the marginal utility of current consumption (current marginal utility), while the right-hand side is the expected value of the marginal utility of future consumption multiplied by β and R (expected marginal utility).¹² Equality between these two terms is a necessary condition for optimizing behavior. If the current marginal utility is higher (lower) than the expected marginal utility, the consumer increases his/her well-being by increasing (decreasing) current consumption. Adjustments of current consumption do not increase the well-being if and only if the equality expressed by equation (20) holds.

To analyze the optimal savings behavior, we set $R = 1.04$, $\beta = 0.96$, $\gamma = 2.00$, and $Y_1 = 1$. Then, we vary Y_2 from 0.50 to 1.50 and p from 1% to 20%. Figure 3 shows the optimal savings for each combination of p and Y_2 . As expected, the optimal savings increases with the probability of unemployment p . Recall that the variance of future income increases with p since it is a direct measure of the risk that consumers face. Furthermore, optimal savings increases as Y_2 decreases. Recall that Y_2 is the expected value of future income, and when the ratio Y_2/Y_1 decreases, it is natural for the consumer to save more.

Figure 3 – Optimal savings: effects of the probability of unemployment and expected future income.



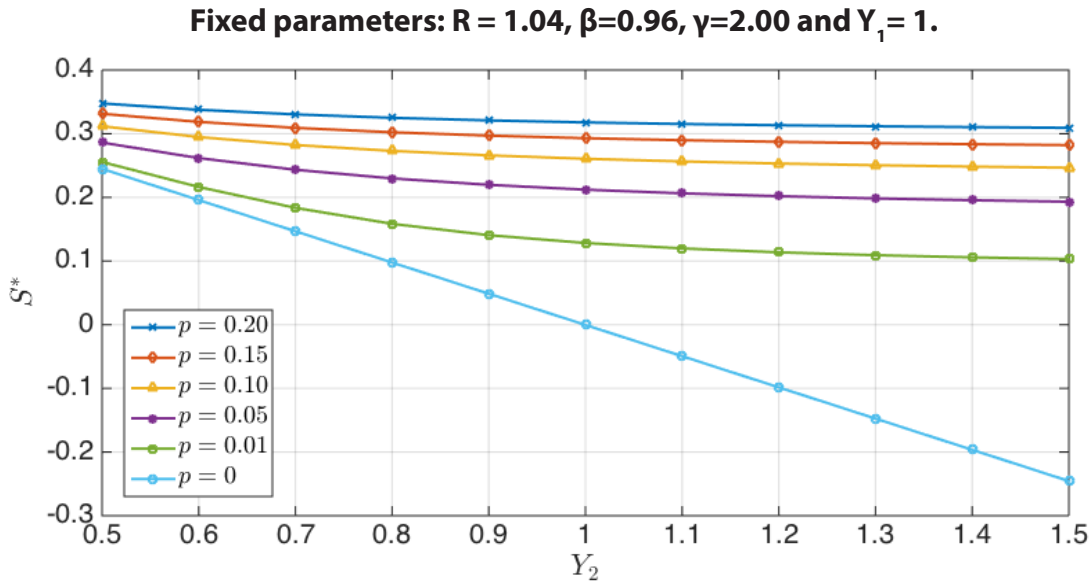
As shown in Figure 3, the optimal savings are positive for each grid point between 0.10 and 0.35. Even when $Y_2 = 2$ and the probability of unemployment is only 1% ($p = 0.01$), the consumer does not borrow. The reason is simple: the risk of unemployment means that the consumer may have no labor income in the second period, and in such a scenario, future consumption can only be financed through positive savings. It is worth mentioning that the model does not have any credit constraints. The consumer could borrow, but he/she chooses not to. Thus, although the motivation is entirely different, consumer behavior resembles that caused by credit constraints.

Precautionary savings are savings that occur in response to uncertainty regarding future income. To show this additional amount of savings, we compare the optimal savings

¹²In all quantitative analyses, we set $\beta = 0.96$ and $R = 1.04$ and consequently, βR is approximately equal to one. In this sense, the consumer chooses the optimal savings by approximating the marginal utility of current consumption and the expectation of the marginal utility of future consumption.

without uncertainty ($p=0$) and the optimal savings with uncertainty ($p>0$).¹³ To be precise, we use the following grid for the unemployment probability: 0%, 1%, 5%, 10%, 15% and 20%. Finally, as in previous exercises, we set $R=1.04$, $\beta=0.96$, $\gamma=2.00$ and $Y_1=1$, with Y_2 and varying from 0.50 to 1.50. Figure 4 displays the results. The optimal savings for $p=0$ has the pattern already discussed in the analysis of Figure 1: *i*) positive when Y_1 is high (relative to Y_2), *ii*) close to zero when Y_1 and Y_2 are similar, and *iii*) negative when Y_1 is low (relative to Y_2). For $p>0$, the consumer faces unemployment risk and savings are always positive. The larger the p , the larger the savings level. By comparing any savings curve for $p>0$ with the curve for $p=0$, we calculate the precautionary savings. Therefore, as expected, precautionary saving is positive and increases when unemployment risk increases.

Figure 4 - Optimum savings: effect of the probability of unemployment and expected future income.



To scrutinize the relevance of precautionary savings, we calculate the precautionary savings as a fraction of the current income, as follows:

$$100 \frac{S_U - S_D}{Y_1} \%, \quad (21)$$

where S_U is the savings level with uncertain income ($p>0$), and S_D is the savings level with determinist income ($p=0$), so $S_D - S_C$ is the precautionary savings. Hence, we calculate the additional non-expenditure fraction of current income due to income risk. We restrict the values for Y_2 to guarantee positive savings even under deterministic income. Table 2 presents the results. It is evident that the percentage of current income allocated to precautionary savings increases with the unemployment probability, which is the measure of the income risk. For instance, for $Y_2=0.50$ and $p=0.01$, precautionary savings represent 1.1% of current income. However, for $Y_2=0.50$ and $p=0.20$, precautionary savings become 10.3% of current income. Furthermore, precautionary savings, as a fraction of current income, increase with the expected income Y_2 . Indeed, when Y_2 increases, the total savings decrease when there is a risk ($p>0$) or not ($p=0$), as detailed in Figure 4. However, the decrease is more accentuated when there is no risk, which explains why precautionary savings (as a fraction of Y_1), increase with Y_2 in Table 2.

¹³When $p=0$, the model becomes that from Section 3.1, in which the optimal savings is given by equation (15).

Table 2 – Precautionary savings: effects of unemployment probability and expected future income**Fixed parameters: $R=1.04$, $\beta=0.96$, $\gamma=2.00$ and $Y_1=1.00$**

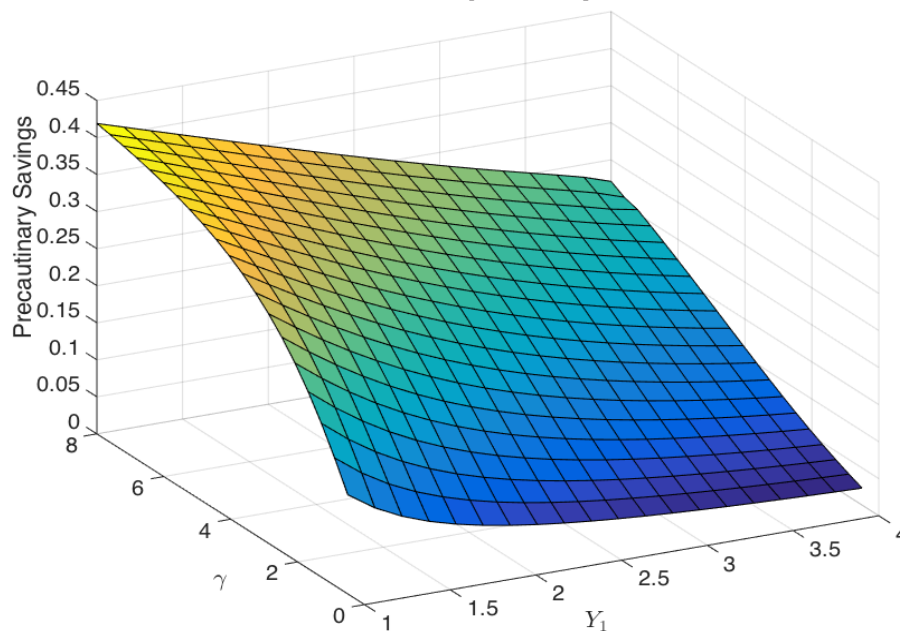
p	Y_2			
	0.50	0.60	0.70	0.80
0.01	1.1%	2.1%	3.7%	6.1%
0.05	4.1%	6.6%	9.7%	13.2%
0.10	6.7%	9.9%	13.6%	17.5%
0.15	8.7%	12.3%	16.2%	20.5%
0.20	10.3%	14.2%	18.4%	22.7%

Finally, we would like to investigate how the coefficient of relative risk aversion and current income affects precautionary savings. As mentioned, the index of relative prudence is equal to one plus the relative risk aversion ($P_R = 1 + \gamma$). However, in all exercises so far, we keep $\gamma=2.00$, which implies that the index of relative prudence is constant and equal to 3.00. Following Kimball's (1990) results, we expect to observe higher precautionary savings when γ is higher. In turn, current income also remains fixed in the previous exercises. However, such income has a critical role. To clarify, note that $E[Y_1 + R^{-1}\tilde{Y}_2] = Y_1 + R^{-1}Y_2$ is the expected present value of the labor income. Therefore, given Y_2 and R , the higher the Y_1 is, the higher the non-riskier share of these resources. For this reason, we expect that increases in Y_1 reduce the precautionary savings, while the other parameters remain fixed.

Figure 5 displays precautionary savings (instead of the total savings) as a function of the coefficient of relative risk aversion and the current income. Note that, by keeping the current income level constant, precautionary savings increase with the relative risk aversion. Hence, the higher the index of relative prudence, the higher the precautionary savings. Moreover, precautionary savings decrease with the current income for a fixed coefficient of relative risk aversion.

Figure 5 - Precautionary Savings: effects of the coefficient of relative risk aversion and current income.

Fixed Parameters: $R=1.04$, $\beta=0.96$, $p=0.10$ and $Y_2=1.00$.



4. A Road Map to Teach the Precautionary Motive

We suggest the following road map for instructors who adopt this material to teach the precautionary motive.

First, apply the topic to students' lives. We suggest the following strategy: 1) ask them how many hours they would study for a final exam if they knew in advance its questions; 2) ask them how many hours they would study for a final exam when there is uncertainty about its questions. The idea is straightforward. If the students knew in advance the questions of the exams, they could study only for these questions. However, due to uncertainty, they tend to study more, covering all topics of the course. This additional effort represents how they react to the uncertainty about the final exam because there is a risk of failure. Analogously, precautionary saving measures how consumers react to future income uncertainty, given the risk of having no income (losing their job).

Once students realize that they react to risk by studying more, go to the second step. Connect this perception with precautionary savings by asking them whether people tend to save more when the unemployment risk increases. Thus, revise the statistic tools and microeconomic concepts of Section 3.1, to help them understand a model that captures the precautionary motive. Make sure they understand why such a revision is necessary. For instance, it is not possible to consider random events, such as unemployment, without the use of random variables.

In the third step, teach the models of Sections 3.2 and 3.3. We prepared an Excel file that solves these models automatically.¹⁴ Use it to solve the models and to discuss how the optimal consumption and savings depend on the structural parameters, such as the relative risk

¹⁴For details, see the Appendix. The file is available at <https://drive.google.com/open?id=1rEf7WdVn1xSJXgOoeiQyL4o5JMbeutWq>

aversion coefficient (γ). At this point, it is worth mentioning a common difficulty of students: to understand that future consumption is a random variable. Although the consumer chooses consumption and savings in the first period, the exact consumption level in the second period is not known in advance. On the one hand, the consumer may lose his/her job, and the second-period consumption is financed only by the financial wealth ("low consumption"). On the other hand, if employed, the second-period consumption is financed by the financial wealth and the second-period income being larger! To help students, the Excel file presents the consumption level in both situations, employed and unemployed. However, in practice, only one of them happens. We suggest that instructors emphasize this feature when using the Excel file.

Finally, the optional fourth step is a critical analysis of the Keynesian consumption function. If the students recognize that the Keynesian function does not capture the precautionary motive, we would have the first evidence of learning. Students can go further by discussing other motives for savings that are not captured by the models in Sections 3.1 and 3.2. After that, the instructor may present the list of nine motives for saving from Section 2. Thus, the students will realize that additional effort is needed to model other motives for saving.

5. Conclusions

This paper presents a list of motives for saving and elaborates on simple models that capture the life-cycle and the precautionary motives. While the former is generally addressed in undergraduate textbooks, either through LCM or PIH, the latter motive is not so present. For this reason, this material constitutes an adequate complement for teaching consumption and saving decisions in undergraduate courses.

The analysis of the precautionary motive depends on knowledge of statistic tools since future income is modeled as a random variable. We assume that either the consumer is employed with a positive income, or he/she is unemployed with no income. It is precisely this uncertainty about future income that leads consumers to change their behavior in relation to the case where future income is deterministic. In our exercises, the consumer reacts to the unemployment risk by increasing their savings. This additional amount of savings is the precautionary savings.

Using a two-period model along with the CRRA utility, we show that precautionary savings increase with the unemployment risk, the relative prudence coefficient, and the future (uncertain) income. Additionally, precautionary savings diminish when current income increases. Although there is no credit constraint in our framework, consumers always save to ensure resources to finance future consumption even if they lose their jobs.

After exposure to this material, we hope that those who were unfamiliar with the precautionary motive will be able to understand it and incorporate it into their analysis of consumer behavior. Indeed, we provide the programs used to generate the figures of this work, allowing the reader to deepen his/her knowledge through the simulation of other scenarios of interest.

Finally, it is worth reinforcing that our analysis is based on the expected utility approach, in which consumers understand the income uncertainty, and the consequent unemployment risk. Furthermore, there is no financial illiteracy, and consumers transfer income from one period to another using interest rate. Therefore, our model does not take into account issues such as financial illiteracy. Indeed, the analysis of savings decisions using the behavioral economics approach have been left for future research.

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Appendix

This paper employs two consumption models to study precautionary savings. The first model assumes that income is deterministic, and we find a closed-form solution for savings described by equation (15). The second model assumes that income is a random variable, and, in this case, we employ a numeric optimization method to find the savings level that solves its Euler equation (see equation (20)). We use the MATLAB Software to solve the second model, and the program used is available upon request.

For undergraduate students, we elaborate an Excel file that solves the consumption models of this paper. The user chooses the values of both the exogenous variables (Y_1 , Y_2 and R) and the parameters (β , γ and ρ), and the Excel spreadsheet automatically calculates the optimal savings for each model. When income is deterministic, the solution comes from equations (11), (12), and (15). When income is uncertain, we employ a Microsoft Excel add-in program, Solver, to solve the Euler equation (20). To find the optimal savings (S^*) using Solver, we input two constraints. First, the consumer cannot save more than the first-period income. Second, he/she cannot borrow so much as $R^{-1} Y_2$. Such debt must be paid in the second period with interest, and it becomes $R R^{-1} Y_2 = Y_2$. Therefore, the second-period income would be used only to pay off the debt $R^{-1} Y_2$, and there would be no resources to finance the second-period consumption. Hence, we know that $-R^{-1} Y_2 < S^* < Y_1$.