JOURNAL OF
ECONOMICS
TEACHING

Roll the Dice: Teaching Expected Value Using Randomly Assigned Grades

In this lesson, students are taught the concept of expected value. Two example problems are provided to teach the concept of expected value. Students are then given the chance to analyze whether to accept a randomly assigned grade. Finally, expected value is used to determine when to enter the Powerball lottery.

Tom Flesher†

†Suffolk County Community College

1. Introduction

Economics educators have bemoaned the lack of evolution in teaching techniques for decades (see, e.g., Asarta et al., 2021; Becker & Watts, 2001). Even so, active learning has been an arrow in the economist's quiver for just as long (Christoffersen, 2002; Simkins, 1999). Active learning can increase student retention and understanding while training students to think like economists (Sankaran et al., 2016).

The exercises presented here help students understand the mathematical concept of expected value, which is key to the microeconomic area of risk and uncertainty, using their grades. Anecdotally, students often have a reductive view of probability as "50-50" – "either it happens or it doesn't" – and helping students understand how expectations are calculated can help them think more clearly in their real-world pursuits. Students who have low mathematical sophistication can practice how to think problems through based on the limited information available to them. All students walk away with the practical knowledge that it is rarely a positive expected value proposition to enter even a large lottery.

The lesson centers around each student rolling a large six-sided die at the beginning of class and recording the result on an index card with his or her name.

2. Direct Instruction and Practice Problems

Teach the formula EV = $\sum_{i=1}^{n} p_i * v_i$. Here, p_i is the probability of outcome *i* and v_i is the value of outcome *i*. Then, remind students of the rule of rational choice: for a risk-neutral agent, a decision is rational if and only if its expected marginal benefit at least covers its expected marginal cost. The lesson plan describes a simple example of a decision about cleaning up after your dog on a walk, which activates students' prior knowledge about the basic mathematics used in this lesson. A second decision, about turnstile-jumping, reinforces the concept and demonstrates to students that the same method can be used to guide policy decisions. The example as written yields a fine of $275; note that the current fine in New York City is $100, and the crime is rarely prosecuted.

3. Discussion of Student Grades

Offer the students a thought experiment: given the chance, would you replace your letter grade in this course with one that is randomly assigned? Tell students that their die rolls in the Do-Now will be used to assign final grades for students who would hypothetically take this risk.

First, determine the average of the randomly assigned grades. Tally students' rolls on the board and then introduce the roll-to-grade correspondence in the lesson plan. Demonstrate that the expected value is 2.67, or a C+ on the 4.0 scale. Calculate the average result for your class, based on the tallied rolls. Depending on the size of your class, this should be close to the expected C+. Use this to reinforce that an expected value is an expectation, not a guarantee.

4. Risk Aversion

Students find the idea of randomly assigned course grades intriguing. Some students are simply risk-loving and enjoy the idea of gambling for a grade. Others might feel the material is difficult or may have many outside obligations. Discussing the costs and benefits of accepting a randomly assigned grade helps students understand that just as not every student rolls the same grade, not every student will make the same decision.

The example to this point asks the students to consider locking in a grade before the end of the semester. Would students be willing to sacrifice the opportunity for a higher grade to avoid the risk of a bad roll? This question is a gentle introduction to the concept of risk aversion. Offer students the opportunity to skip the die roll and the remainder of the course, but limit them to a grade below the expectation – would they do it for a C? Would they go as low as a D+? If time permits, the St. Petersburg Paradox example can be used to deepen understanding.

5. The Powerball Activity

A closing activity centered on the Powerball lottery helps students practice mathematical skills and work with the concept of expected value.

The Powerball lottery is a multi-state game of chance in which six numbers – five "regular" numbers plus a sixth "Powerball" number – are drawn. Players pay $2 and choose six numbers to play. Twice each week, six numbers are selected in a televised drawing. Players win prizes for matching the Powerball number alone or several of the numbers with or without the Powerball number. The included handout details these prizes, including the value *v*, the probability *p*, and the value multiplied by the probability of winning it (*p**v*). Matching all five numbers plus the Powerball wins a grand prize that grows in value each time the previous drawing did not have a grand prize winner. Students find it tempting to try to beat this game of chance and are generally interested to learn that when the grand prize grows large enough the expected value of a Powerball ticket is greater than the ticket's price.

The price of a ticket is $2. Multiplying the probability of each fixed-value prize by its payoff and adding the products yields an expected value of 32 cents. The probability of winning the grand prize is 0.0000000034, so the expected value calculation for a jackpot of value *v* is:

$$\$2 < .32 + 0.0000000034 * v$$

This is equivalent to

$$\$1.68 < 0.0000000034 * v$$

so $v > \$494{,}117{,}647.06$. Elicit from students that this is about half a billion dollars, and if the government takes half the jackpot in taxes, the pot must be about a billion dollars to make it rational for a risk-neutral player to buy a Powerball ticket. This lesson is especially timely and popular about once a semester when the Powerball jackpot reaches this level.

References

Asarta, C. J., Chambers, R. G., & Harter, C. 2021. Teaching methods in undergraduate introductory economics courses: Results from a sixth national quinquennial survey. *The American Economist, 66,* 18-28. DOI: 10.1177/0569434520974658

Becker, W. E., & Watts, M. 2001. Teaching economics at the start of the 21st century: Still Chalk-and-Talk. *American Economic Review, 91*, 446-451. DOI: 10.1257/aer.91.2.446

Christoffersen, S. 2002. An active learning tool for the principles of economics: The allocation exercise. *The American Economist, 46,* 65-68. DOI: 10.1177/056943450204600206

Sankaran, C., Mulroney, T., & Corcoran, D. 2016. An exercise on creating a student expenditure basket. *Journal of Economics Teaching, 2,* 90-110. DOI: 10.58311/jeconteach/2cf069c828d965e2da60f9bf2c4778ac1170f48f

Simkins, S. P. 1999. Promoting active-student learning using the world wide web in economics courses. *Journal of Economic Education, 30,* 278-287. DOI: 10.2307/1183067

LESSON PLAN FOR "ROLL THE DICE"

OBJECTIVE: Students will be able to use the tool of expected value to compare options under uncertainty. This lesson also reiterates the key concept of rational choice using marginal benefit and marginal cost.

MATERIALS: Dice (large yard dice work best; I use the same ones I use for Monte Carlo quizzing); index cards; whiteboard or chalkboard.

DO NOW: As students enter the room, ask each one to roll a six-sided die and write the number and their name on an index card.

INTRODUCTION: (10 minutes) Make any announcements; as appropriate to the point in the semester, ask students if they are comfortable with their grades. Discuss why students' expectations about grades might or might not have been accurate. Introduce the idea that you are going to offer the students a chance to replace their course grade with one that is randomly-assigned, then collect index cards.

DIRECT INSTRUCTION: (10 minutes) Teach the formula EV = $\sum_{i=1}^{n} p_i * v_i$ for all outcomes.

Making it clear this is a hypothetical, offer to give students a fresh start, but there's a catch: You can choose, today, to roll a die [as they already have done] and replace the course grade they would earn based on the syllabus grading policy with one assigned by chance.

Before writing the data on the board, introduce this scale:

If you rolled a 6, you get an A.

If you rolled a 5, you get an A. (Elicit from students that this seems skewed in their favor.)

If you rolled a 4, you get a B.

If you rolled a 3, you get a C.

If you rolled a 2, you get a D.

If you rolled a 1, you get an F.

Elicit or remind students of the 4.0 grading scale: A = 4.0, B = 3.0, C = 2.0, D = 1.0, F = 0.0. Ask them how good a deal the random calculation is – answers will vary.

Ask the students to calculate the expected value of rolling a die (essentially the grade to be expected from choosing to randomly assign a grade instead of accepting the grade earned). After giving them a few minutes to do so, demonstrate the calculation yourself. [1/6*(0+1+2+3+4+4) = 14/6 = 2.67, or a C+]

Key points to emphasize include that no one will actually "roll" a C+ and that even with this average some students will do better and some students will do worse.

COLLECT DATA: (10 minutes) Tally the rolled numbers on the board. Calculate the average roll, which will likely be close to 2.67.

RATIONAL CHOICE: (25 minutes) Remind students of the rule of rational choice: that a decision is considered rational as long as its marginal benefit at least covers its marginal cost. Offer two examples:

1. When I lived in Farmingdale, the fine for not picking up after my dog was $25. (That was exceedingly low.) Is it rational to pick up after my dog? Let's use rational choice theory to make this decision

    a. How much would you pay to avoid picking up after your dog? [One student answered $100, but you can work with any answer.]

    b. What do you think is the probability that you will be caught and fined? [Students often say it's 50-50, but you can work with any answer.]

    c. Marginal Benefit = 100 [avoiding the unpleasant task]

    d. Marginal Cost = Expected fine = Probability of fine*Size of fine = .5*25

    e. 100 > .5*25, so it is rational not to pick up after your dog.

    f. Alternate calculation: Say students said they would pay $5 to avoid picking up after their dog and there is a 30% chance of being fined:

        5 < .3*25, so it is rational to pick up after your dog.

    Emphasize to the students that they are not actually paying to avoid picking up after the dog, using this opportunity to remind them that we care about their utility, not simply the money paid or earned.

2. The subway costs $2.75 to ride. If you have a 1% chance of getting caught, how high should the fine be to deter turnstile-jumping?

    a. Marginal benefit of turnstile-jumping = 2.75 [fee saved]

    b. Marginal Cost = Expected fine = Probability of fine*Size of fine

        i. .01*x

    c. 2.75 = .01x (if you are indifferent between jumping or paying to ride)

    d. Then x = 275, so a fine of at least $275 should deter turnstile-jumping.

    e. Currently, the fine is $100, and Manhattan DAs generally don't bother prosecuting it.

Now, calculate the average grade for your class. Highlight that it should be close to 2.67. Ask students to compare their current course grade to the one that they rolled: who would have done better than their current grade and who would have done worse?

Remind students of MB = MC and ask what sort of student would take the chance of rolling the die today: students who expect a grade less than C+, or students who have high opportunity costs (e.g. new parents or students taking very heavy course loads).

RISK AVERSION: (10 minutes) Introduce the idea of a certainty equivalent: what grade would you accept to simply forego the die roll *and* the final exam?

Some students are likely to request a higher grade – e.g., "I would only do this if it would guarantee me an A." This reply can be used as a jumping-off point to rank the uncertainty associated with each option. The students can be guided to rank the die roll as the riskiest outcome since even though it has the highest potential value it can also result in failure. If necessary, remind the class that students who do the assigned work will rarely fail the course. The fixed grade is the lowest-risk option since its outcome is guaranteed. Taking the course normally and having it evaluated for a grade is a moderate-risk option. Thus, students who prefer the die roll to a fixed grade value risk.

If you are working with students who have the requisite mathematical sophistication, this is a good time to refer to variance – note that as a heuristic, the die roll seems the most uncertain, the traditional grading method modestly uncertain, and the fixed grade completely certain, so we would expect to see the highest variance with the die roll and the lowest with the fixed grade.

Many students can thus be persuaded to agree that, if a person is risk-averse, they would be willing to accept a grade lower than even the expected C+. Elicit that "Cs get degrees," and (at least at the school where this lesson was developed) "D stands for 'done'" (in that a course graded D can fulfill a requirement and need not be repeated). Ask: Who would be willing to take a C if it meant you could leave the course? Presumably, some will agree. Then, ask: Would anyone be willing to accept a D? Perhaps some will. Although students will likely characterize this as a time savings, steer the discussion to risk: would you want to spend all the time to prepare for this course's requirements if you weren't sure you'd pass?

At the level this lesson is designed for, no further mathematics are appropriate, but emphasize the takeaway that many students are in reality risk-neutral and would be willing to accept a lower-than-expected grade if it limits the chance that they will fail the course. Depending on the instructor's rapport with the students, it may be fun to ask if students might bribe the instructor for certainty and use the money values to discuss the conversion of money to utility.

FLEX ACTIVITY: (10 minutes) If time permits, describe the St. Petersburg paradox in the context of the course: how much would you pay for the following offer?

You flip a coin. The first time you flip, the flip is worth 2 points. The second is worth 4 points. The third is worth 8 points, the fourth 16 points, and so on. If the coin lands on Tails, you flip again. As soon as the coin lands on Heads, your game ends and you collect the extra credit points you have earned at that flip (so if you flip two tails and a head, you get 8 points of extra credit).

Elicit some offers – most students will not offer very much money. Calculate the expected value: it is infinite! Discuss with the students why they are not willing to offer infinite money for this opportunity even though they stand to gain significantly.

It can be enlightening to ask students to compare this value to the amount they would hypothetically pay the instructor as a bribe in exchange for an A, since this game yields an automatic A as its expected value but students are unlikely to be willing to pay as much to play as they would pay for an A outright. Use this to emphasize the value of certainty.

PRACTICE: (20 minutes) Have students use the MB = MC equation and the attached handout to calculate the Powerball jackpot size necessary to make it rational to buy a ticket.

Match	Prize	Odds: 1 in	Probability	*p*v*
5+P	GP	292,201,338.00	0.0000000034	
5	$ 1,000,000.00	11,688,053.52	0.0000000856	0.09
4+P	$ 50,000.00	913,129.18	0.0000010951	0.05
4	$ 100.00	36,525.17	0.0000273784	0.00
3+P	$ 100.00	14,494.11	0.0000689935	0.01
3	$ 7.00	579.76	0.0017248517	0.01
2+P	$ 7.00	701.33	0.0014258623	0.01
1+P	$ 4.00	91.98	0.0108719287	0.04
P	$ 4.00	38.32	0.0260960334	0.10
			Sum	0.32