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On Real Business Cycle Models in Excel

One of the biggest challenges of teaching macroeconomics to undergraduate students is bridging the gap between intermediate and modern micro-founded macroeconomic models. Model simulations help by allowing students to experiment with the model results without fully solving the model. Some modern macroeconomic models can be simulated in Excel, accommodating students with a limited programming background. For example, Hokari et al. (2007) simulate a simple real business cycle model in Excel. I correct their simulation method, provide a model simulation workbook, and provide a lesson plan to evaluate how well the real business cycle model fits business cycle facts.

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1. Introduction

A longstanding problem when teaching an advanced undergraduate macroeconomics class is bridging the gap between models traditionally taught in intermediate macroeconomics and modern micro-founded models without overwhelming students with technical details (Erekson, Raynold, & Salemi, 1996). For example, real business cycle (RBC) models explain macroeconomic fluctuations as the result of economic agents maximizing their lifetime utility subject to resource and production constraints. This emphasis on micro-foundations is in sharp contrast to models traditionally taught in intermediate macroeconomics, such as the IS/LM model, but the understanding of micro-foundations aids in understanding of other modern macroeconomic models (Froyen, 1996).

Model simulation is one tool that makes macroeconomic models more accessible to those without a technical background. While many simulations are performed in MATLAB or other software that require additional coding experience, Hokari et al. (2007) simulate a simple RBC in Excel with no add-ins. While this simulation makes the model accessible to students with little technical knowledge, several errors in the published solution stymie its use. In this paper, I describe the model and provide a corrected Excel simulation. The RBC model allows economists to study business cycles using a micro-founded dynamic stochastic general equilibrium (DSGE) model that generates simulated macroeconomic data comparable to actual data (Rebelo, 2005). I also provide a lesson plan to guide students through a simulation of the real business cycle model to evaluate how it fits business cycle facts. The lesson plan also includes examples of one-time and permanent technology shocks, as well as the effects of a one-time capital loss, such as from natural disasters.

This paper contributes to the literature on the modernization of the undergraduate macroeconomics curriculum through integrating DSGE models. The model and lesson plan are useful for undergraduate students who can work with the Excel simulation without programming or extensive mathematical preparation, and for advanced undergraduate and master's students who can also derive the solution of the approximation. They can be used alongside intermediate macroeconomics textbooks and curricula that include real business cycle models, such as Williamson (2018), and Garín, Lester, and Sims (2021). While Barreto (2016), and Bongers, Gómez, and Torres (2021) provide Excel simulations of several macroeconomic models, including some dynamic models in the latter case, this simulation and lesson plan demonstrate the fit of the RBC model to business cycle facts. This fit is important, because it has been a key metric for evaluating modern business cycle models (Kehoe, Midrigan, & Pastorino, 2018). The lesson plan includes a definition sheet to introduce students to methods for finding business cycle facts and evaluating the model's fit to them, including detrending data with the Hodrick-Prescott (1997) filter, the first order autocorrelation of variables, and the relative standard deviation of variables.

This paper introduces the model and lesson plan. The lesson plan and Excel simulation book are included as supplementary materials. Section 2 of this paper introduces the RBC model and the Excel simulation, while the full approximation details, including corrections to Hokari et al. (2007), are in the Appendix. Section 3 discusses the lesson plan and student worksheet, and Section 4 concludes the paper.

2. Model and Method in Excel

Hokari et al. (2007) approximate the solution to a simple real business cycle model by linearizing around the model's steady state and then using a method by Farmer (1999) to solve the resulting system of linearized equations. The model is given below, and the details of the solution method are in the Appendix.

Output in the economy (y_t) follows a Cobb-Douglas production function in capital (k_t) and labor (l_t):

(1) $$y_t = z_t k_t^{\alpha} l_t^{(1-\alpha)}, 0 < \alpha < 1.$$

Productivity z_t follows a stochastic process defined by:

(2) $$\log(z_t) = \rho \log(z_{t-1}) + \epsilon_t, 0 < \rho < 1.$$

The productivity shock, ϵ_t, is an identically distributed shock drawn at the beginning of each period $t(t = 1, \ldots, \infty)$ that follows $\epsilon_t \sim N(0, \sigma^2)$. Initial capital (k_0) and initial productivity (ϵ_0) are known.

Hokari et al. (2007) solve a social planner's problem for a plan that maximizes the expected lifetime utility evaluated at time 0 of people in the economy:

(3) $$E_0\left[\beta^t \sum_{t=0}^{\infty} [\log(c_t) + \gamma \log(1 - l_t)]\right]$$

subject to:

(4) $$c_t + k_{t+1} = y_t + (1 - \delta)k_t.$$

Consumption in time t is c_t, the discount factor β is between 0 and 1, and $\gamma > 0$. Their method approximates the model's solution by linearizing it around its steady state. The corrected solution, summarized by equations (A12)-(A17) in the Appendix, expresses the deviation of model's control variables c_t, l_t, k_{t+1} and period $t + 1$ shock z_{t+1} from their steady state as a linear function of the deviation model's state variables from their steady state, where variables with a hat are the fraction deviation of each variable from its steady state (eg. $\widehat{c_t} = \frac{c_t - \bar{c}}{\bar{c}}$). Assuming the economy is at its steady state at time $t = 0$, the solution allows simulation of impulse responses of model variables with a productivity shock at time $t = 1$ and an economy simulation with productivity shocks in each period by simulating $\epsilon_t(t = 1, \ldots, \infty)$.

The simulation in Excel is constructed by first inputting the values of model parameters in an Excel workbook (cells B2-B8 in the workbook in the supplementary materials). Model parameters are set at the same values as in King and Rebelo (1999) $(\alpha = 0.333, \beta = 0.984, \delta = 3.48, \rho = 0.979, \sigma = 0.0072)$. Second, the steady-state values of macroeconomic variables are calculated from the parameter values (cells D3-D11). Third, the individual entries of A, M, Q, and λ from the model's solution are computed from the other values (rows 13-25 in the workbook). Fourth, if simulating the model with productivity shocks each period, the shocks are drawn starting at $t = 1$ (column K of the "Economy Simulation Calculations" sheet starting at row 31)[1]. Fifth, initial time 0 capital, and all other variables, are

[1] Two issues with the default Excel random number functions are that they lack a fixed seed and generate new draws whenever the sheet updates. I use the Park-Miller (1988) linear congruential random number generator instead of these functions to ensure that the results are replicable given an initial seed. The algorithm generates a sequence of random numbers $J_t, t = 1, \ldots, T$ using $J_{t+1} = aJ_t \bmod m$ where a is the multiplier and m is the modulus. I use common values $a = 48,271$ and $m = 2^{31} - 1 = 2,147,483,647$. The values J_t can be converted to draw from an approximately uniform distributed random variable that I define as j_t by dividing J_t by the modulus $j_t = \frac{X_t}{2^{31}-1}$.

set at their steady-state levels (row 30). Finally, deviations of model variables from their steady state at each t are calculated (columns C-J starting at row 31) for each of the 1,000 periods of the simulation. The simulation results graphs display the deviation of macroeconomic variables for the first 100 periods.

3. Lesson Plan and Worksheet

The lesson plan in the supplemental material guides students to examine how well the real business model explains business cycle facts using a simulation of the RBC model from the Excel workbook. Its intended audience is advanced undergraduate or graduate macroeconomics students who would like to work with the RBC model simulation without needing substantial programming experience. Substantial mathematical preparation is not necessary to understand the simulation results, but advanced undergraduate or master's students with preparation in calculus, dynamic optimization, and linear algebra would benefit from the details of the approximation in the Appendix.

The goal of this lesson is to show how the RBC model fits facts about business cycles. Instructors introducing the RBC model should use the model in Section 2 of the main text as a basic example. Then, they need to introduce business cycle facts from the RBC literature as studied using the Hodrick-Prescott (1997) filter. The filter decomposes time series data into a smooth trend component and a cyclical component. Isolating the cyclical component of macroeconomic time series allowed them to study the characteristics of business cycles and the relationships among macroeconomic variables. As summarized by Rebelo (2005), the most important of these facts are:

1. Investment is about 3 times more volatile than output.
2. Nondurable consumption is less volatile than output.
3. Hours worked and output are similarly volatile.
4. Almost all macroeconomic variables are strongly procyclical.
5. There is substantial persistence in macroeconomic variables.

Students simulate the RBC model and then evaluate the simulation's ability to fit the above business cycle facts through results' relative standard deviation, correlation between variables, and the first order autocorrelation of variables. The definition sheet includes information about each of these statistics, a brief description of the Hodrick-Prescott filter used to detrend the macroeconomic data before determining the business cycle facts, and a description of procyclical and countercyclical economic variables.

While the typical simulation captures most of these facts, it may fail to generate sufficient volatility in hours, which instructors could use to motivate the discussion of the indivisible labor extension of Hansen (1985) where this volatility is a central issue.

The lesson plan also includes three shocks and changes that can be modeled using impulse responses shown on the Impulse Response Main Results sheet. First, the sheet's default setting is a temporary negative 1% shock to productivity. This shock illustrates the framework's explanation of recessions as the efficient response of individuals and households to a disruption, such as bad weather, supply disruptions, or regulatory changes, that temporarily hurts productivity. Recessions in this model are deteriorations in productive capacity, not failures of demand or financial disruptions, and stabilization policies in simple extensions are inefficient since firms and households optimally respond to unavoidable losses without stabilization. Second, a one-time capital loss can be modeled on the Impulse Response Main Sheet by adjusting the initial capital stock in the Impulse Response Calculations sheet. A capital loss is a common method to model the effects of a natural disaster on the economy (Hallegatte & Ghil, 2008). The capital loss simulation shows that the disaster stimulates investment and output, while reducing consumption, during the recovery back to the economy's steady state. Third, immediate or gradual increases in technology that lead to a permanent increase in productivity can be modeled through adjustments to the productivity shock or productivity in the Impulse Response Calculations sheet. This exercise shows the pattern of increased investment that occurs as the economy transitions to the new steady state.

It may seem that students without strong mathematical preparation would struggle with the simulation because of the technical level of the proof, but I have not found that to be the case. I teach this RBC model in an advanced undergraduate macroeconomics class that has prerequisites of intermediate macroeconomics and one semester of calculus. The goal of the class is to explore macroeconomic schools of thought and their views on monetary and fiscal policies with the IS/LM model from intermediate macroeconomics as a starting point. I do not believe it is wise to proceed directly from the IS/LM model to this RBC model since the simulation approach by itself would leave out important intuition and concepts needed to understand the results. Students need to understand the concepts of dynamic models and steady states, but these concepts do not need micro-foundations to be introduced. For example, the Hansen-Samuelson multiplier-accelerator model (Samuelson, 1939) and the Cagan (1956) hyperinflation model can introduce students to dynamic models, stability, impulse response diagrams, and steady states. The Fisher two-period model can develop intuition for intertemporal substitution. The RBC model was students' introduction to micro-founded macroeconomic models. Solis-Garcia (2018) describes successful teaching of undergraduate students using a variety of DSGE models with the RBC model as a base since many models used to study labor markets, financial frictions, and fiscal policy can be thought of as RBC models with extra features.

Critics of real business cycle theory point out that it relies on large productivity disturbances to explain fluctuations in employment, relies on the intertemporal substitution of leisure to explain employment changes, and trivializes the social cost of economic fluctuations (Mankiw, 1989). However, its numerous extensions address its shortcomings and many important questions that would motivate additional classroom examples and discussion. For example, Merz (1995) and Andolfatto (1996) incorporate labor market search into the RBC model, which introduces involuntary unemployment and improves its fit to labor market facts. A discussion of sticky prices and sticky wages could lead to New Keynesian models where monetary policy can stabilize the economy. McGrattan (1994) evaluates the welfare costs of labor and capital taxation in an RBC model with fiscal policy. Backus, Kehoe, and Kydland (1992) extend the RBC model to a two-country model to study the correlation of macroeconomic variables across countries. Their finding that the actual correlation of consumption across countries is lower than the correlation of output compared to the model's results was identified as a major puzzle in international economics by Obstfeld and Rogoff (2000). Kehoe, Midrigan, and Pastorino (2018) show that a modern business cycle model, in the form of a simple real business cycle model with financial and labor market frictions, can account for the fluctuations in the macroeconomy observed during the Great Recession.

4. Conclusion

Simulations of advanced macroeconomic models, such as the real business cycle model, that are accessible to students help make macroeconomic education adaptable to new developments, which helps modernize the undergraduate curriculum (Erekson, Raynold, & Salemi, 1996). The corrected results show that the real business cycle model can be simulated in Excel without macros or add-ins. The lesson plan guides students to use the model to study its fit to business cycle facts and to model the effects of productivity shocks, technology improvements, and capital losses from events like natural disasters.

References

Andolfatto, D. (1996). Business cycles and labor-market search. *American Economic Review*, 86(1), 112-132. https://www.jstor.org/stable/2118258

Backus, D. K., Kehoe, P. J., & Kydland, F. E. (1992). International real business cycles. *Journal of Political Economy*, 100(4), 745-775. DOI: 10.1086/261838

Barreto, H. (2016). Teaching Macroeconomics with Microsoft Excel®. Cambridge University Press.

Bongers, A., Gómez, T., & Torres, J.L. (2021). Dynamic Macroeconomic Models with Excel. *Journal of Economic Education*, 52(4), 372. DOI: 10.1080/00220485.2021.1963365

Cagan, P. (1956). The monetary dynamics of hyperinflation. In M. Friedman (Ed.), *Studies in the Quantity Theory of Money* (pp. 25–117), University of Chicago Press.

Erekson, O. H., Raynold, P., & Salemi, M. K. (1996). Pedagogical issues in teaching macroeconomics. *The Journal of Economic Education*, 27(2), 100-107. DOI: 10.1080/00220485.1996.10844899

Farmer, R. E. (1999). *The macroeconomics of self-fulfilling prophecies*. MIT Press.

Froyen, R. T. (1996). The evolution of macroeconomic theory and implications for teaching intermediate macroeconomics. *The Journal of Economic Education*, 27(2), 108-115. DOI: 10.1080/00220485.1996.10844900

Garín, J., Lester, R., & Sims, E. (2021). *Intermediate Macroeconomics* (version 3.0.1). https://juliogarin.com/files/textbook/GLS_Intermediate_Macro.pdf

Hallegatte, S., & Ghil, M. (2008). Natural disasters impacting a macroeconomic model with endogenous dynamics. *Ecological Economics*, 68(1-2), 582-592. DOI: 10.1016/j.ecolecon.2008.05.022

Hansen, G. D. (1985). Indivisible labor and the business cycle. *Journal of Monetary Economics*, 16(3), 309-327. DOI: 10.1016/0304-3932(85)90039-X

Hodrick, R. J., & Prescott, E. C. (1997). Postwar US business cycles: An empirical investigation. Journal of Money, Credit, and Banking, 29(1), 1-16. DOI: 10.2307/2953682

Hokari, T., Iimura, M., Murakoshi, S., & Onuma, Y. (2007). Simulating a simple real business cycle model using Excel. *Computers in Higher Education Economics Review*, 19(1), 16-20.

Kehoe, P. J., Midrigan, V., & Pastorino, E. (2018). Evolution of modern business cycle models: Accounting for the great recession. *Journal of Economic Perspectives*, 32(3), 141-166. DOI: 10.1257/jep.32.3.141

King, R. G., & Rebelo, S. T. (1999). Resuscitating real business cycles. *Handbook of Macroeconomics*, 1, 927-1007. DOI: 10.1016/S1574-0048(99)10022-3

Mankiw, N. G. (1989). Real business cycles: A new Keynesian perspective. *Journal of Economic Perspectives*, 3(3), 79-90. DOI: 10.1257/jep.3.3.79

McGrattan, E. R. (1994). The macroeconomic effects of distortionary taxation. *Journal of Monetary Economics*, 33(3), 573-601. DOI: 10.1016/0304-3932(94)90044-2

Merz, M. (1995). Search in the labor market and the real business cycle. *Journal of Monetary Economics*, 36(2), 269-300. DOI: 10.1016/0304-3932(95)01216-8

Obstfeld, M., & Rogoff, K. (2000). The six major puzzles in international macroeconomics: Is there a common cause? *NBER Macroeconomics Annual*, 15, 339-390. DOI: 10.1086/654423

Park, S. K., & Miller, K. W. (1988). Random number generators: Good ones are hard to find. *Communications of the ACM*, 31(10), 1192-1201. DOI: 10.1145/63039.63042

Rebelo, S. (2005). Real business cycle models: Past, present, and future. *Scandinavian Journal of Economics*, 107(2), 217-238. DOI: 10.1111/j.1467-9442.2005.00405.x

Samuelson, P. A. (1939). Interactions between the multiplier analysis and the principle of acceleration. *The Review of Economics and Statistics*, 21(2), 75-78. DOI: 10.2307/1927758

Solis-Garcia, M. (2018). The macro pedagogy debate: Teaching DSGE to undergraduates symposium: Yes we can! Teaching DSGE models to undergraduate students. *The Journal of Economic Education*, 49(3), 226-236. DOI: 10.1080/00220485.2018.1464987

Williamson, S. (2018). *Macroeconomics*. Pearson.

Appendix A: Calculations for Approximation of Model Solution

This section provides details of the solution method provided by Hokari et al. (2007) and corrections for their solution. The details are not necessary for undergraduate students who study the simulations, but they would be of interest to advanced undergraduate and master's students learning about model solution methods.

The first order conditions of the social planner problem (3)-(4) are given by (A1)-(A4):

(A1) $$\frac{1}{c_t} = \beta E_t\left[\frac{\alpha z_{t+1}\left(\frac{k_{t+1}}{l_{t+1}}\right)^{\alpha-1}+1-\delta}{c_{t+1}}\right]$$

(A2) $$c_t + k_{t+1} = z_t\left(\frac{k_t}{l_t}\right)^{\alpha} l_t + (1-\delta)k_t$$

(A3) $$\frac{\gamma}{1-l_t} = \frac{(1-\alpha)z_t\left(\frac{k_t}{l_t}\right)^{\alpha}}{c_t}$$

(A4) $$\log(z_{t+1}) = \rho\log(z_t) + \varepsilon_{t+1}.$$

These conditions lead to the steady state around which they then linearize:

(A5) $$\frac{\bar{k}}{\bar{l}} = \left(\frac{\alpha}{\frac{1}{\beta}-1+\delta}\right)^{\frac{1}{1-\alpha}}$$

(A6) $$\bar{c} = \frac{\left(1-\alpha\right)\left(\frac{\bar{k}}{\bar{l}}\right)^{\alpha}\left(1-\bar{l}\right)}{\gamma}$$

(A7) $$\bar{k} = \frac{\left(\frac{\bar{k}}{\bar{l}}\right)^{\alpha}-\bar{c}}{\delta}$$

(A8) $$\bar{z} = 1$$

A variable with a bar is the steady state of that variable, and a variable with a hat is the fraction deviation of that variable from the steady state (e.g. $\widehat{z_t} = \frac{z_t-\bar{z}}{\bar{z}}$).

The authors proceed with their linearization. First, the authors linearize using (A1)-(A4) around the steady state (A5)-(A8). They find that labor can be solved in terms of the other variables:

(A9) $$\widehat{l_t} = \frac{-\tau\widehat{c_t}+\alpha\tau\widehat{k_t}+\tau\widehat{z_t}}{\alpha\tau-\frac{\gamma\bar{l}}{\left(1-\bar{l}\right)^2}},$$

$$\text{where } \tau = \frac{\left(1-\alpha\right)\bar{z}\left(\frac{\bar{k}}{\bar{l}}\right)^{\alpha}}{\bar{c}}$$

I define τ above to simplify the notation of (A9). They write the linearized system of equations describing the dynamic relationship between consumption, capital, and productivity as:

(A10)
$$\begin{bmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix} \begin{bmatrix} \hat{c}_t \\ \hat{k}_t \\ \hat{z}_t \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ 0 & b_{22} & 0 \\ 0 & 0 & b_{33} \end{bmatrix} \begin{bmatrix} \hat{c}_{t+1} \\ \hat{k}_{t+1} \\ \hat{z}_{t+1} \end{bmatrix} + \begin{bmatrix} 0 & b_{11} & b_{13} \\ 0 & 0 & 0 \\ 1 & 0 & b_{33} \end{bmatrix} \begin{bmatrix} \epsilon_{t+1} \\ E_t(\hat{c}_{t+1}) - \hat{c}_{t+1} \\ E_t(\hat{z}_{t+1}) - \hat{z}_{t+1} \end{bmatrix}.$$

$$A \begin{bmatrix} \widehat{c_t} \\ \widehat{k_t} \\ \widehat{z_t} \end{bmatrix} = B \begin{bmatrix} \widehat{c_{t+1}} \\ \widehat{k_{t+1}} \\ \widehat{z_{t+1}} \end{bmatrix} + C \begin{bmatrix} \epsilon_{t+1} \\ E_t\left(\widehat{c_{t+1}}\right) - \widehat{c_{t+1}} \\ E_t\left(\widehat{z_{t+1}}\right) - \widehat{z_{t+1}} \end{bmatrix}.$$

Then, the values in A and B are:

$$a_{11} = -1/\bar{c}$$

$$a_{21} = \bar{c} + \frac{\left[\left(1-\alpha\right)\bar{z}\,\bar{l}\left(\frac{\bar{k}}{\bar{l}}\right)^{\alpha}\right]}{\frac{\alpha\left(1-\alpha\right)\bar{z}\left(\frac{\bar{k}}{\bar{l}}\right)^{\alpha}}{\bar{c}} - \frac{\gamma\bar{l}}{\left(1-\bar{l}\right)^{2}}} \frac{\left(1-\alpha\right)\bar{z}\left(\frac{\bar{k}}{\bar{l}}\right)^{\alpha}}{\bar{c}}$$

$$a_{22} = -\frac{\bar{k}}{\beta} - \frac{\left(1-\alpha\right)\alpha\bar{z}\,\bar{l}\left(\frac{\bar{k}}{\bar{l}}\right)^{\alpha}}{\frac{\alpha\left(1-\alpha\right)\bar{z}\left(\frac{\bar{k}}{\bar{l}}\right)^{\alpha}}{\bar{c}} - \frac{\gamma\bar{l}}{\left(1-\bar{l}\right)^{2}}} \frac{\left[\left(1-\alpha\right)\bar{z}\left(\frac{\bar{k}}{\bar{l}}\right)^{\alpha}\right]}{\bar{c}}$$

$$a_{23} = \frac{\bar{c}\,\bar{l}}{\frac{\alpha\left(1-\alpha\right)\bar{z}\left(\frac{\bar{k}}{\bar{l}}\right)^{\alpha}}{\bar{c}} - \frac{\gamma\bar{l}}{\left(1-\bar{l}\right)^{2}}} \left(\frac{\left(1-\alpha\right)\bar{z}\left(\frac{\bar{k}}{\bar{l}}\right)^{\alpha}}{\bar{c}}\right)^{2} - \bar{z}\bar{k}^{\alpha}\bar{l}^{1-\alpha}$$

$$a_{33} = [-\rho]$$

$$b_{11} = -\frac{\beta\left(1-\delta+\alpha\bar{z}\left(\frac{\bar{k}}{\bar{l}}\right)^{\alpha-1}\right)}{\bar{c}} - \left[\frac{\frac{\beta\left(1-\alpha\right)\alpha\bar{z}\left(\frac{\bar{k}}{\bar{l}}\right)^{\alpha-1}}{\bar{c}}}{\frac{\alpha\left(1-\alpha\right)\bar{z}\left(\frac{\bar{k}}{\bar{l}}\right)^{\alpha}}{\bar{c}} - \frac{\gamma\bar{l}}{\left(1-\bar{l}\right)^{2}}} \frac{\left(1-\alpha\right)\bar{z}\left(\frac{\bar{k}}{\bar{l}}\right)^{\alpha}}{\bar{c}}\right]$$

$$b_{12} = \frac{\beta\left(\alpha-1\right)\alpha\bar{z}\left(\frac{\bar{k}}{\bar{l}}\right)^{\alpha-1}}{\bar{c}} - \left[\frac{\frac{\beta\left(\alpha-1\right)\alpha\bar{z}\left(\frac{\bar{k}}{\bar{l}}\right)^{\alpha-1}}{\bar{c}}}{\frac{\alpha\left(1-\alpha\right)\bar{z}\left(\frac{\bar{k}}{\bar{l}}\right)^{\alpha}}{\bar{c}} - \frac{\gamma\bar{l}}{\left(1-\bar{l}\right)^{2}}} \frac{\alpha\left(1-\alpha\right)\bar{z}\left(\frac{\bar{k}}{\bar{l}}\right)^{\alpha}}{\bar{c}}\right]$$

$$b_{13} = \frac{\beta\alpha\bar{z}\left(\frac{\bar{k}}{\bar{l}}\right)^{\alpha-1}}{\bar{c}} + \frac{\frac{\beta\left(1-\alpha\right)\alpha\bar{z}\left(\frac{\bar{k}}{\bar{l}}\right)^{\alpha-1}}{\bar{c}}}{\frac{\alpha\left(1-\alpha\right)\bar{z}\left(\frac{\bar{k}}{\bar{l}}\right)^{\alpha}}{\bar{c}} - \frac{\gamma\bar{l}}{\left(1-\bar{l}\right)^{2}}} \frac{\left(1-\alpha\right)\bar{z}\left(\frac{\bar{k}}{\bar{l}}\right)^{\alpha}}{\bar{c}}$$

$$b_{22} = -\bar{k}$$

$$b_{33} = -1$$

In Hokari et al., $a_{21}, a_{22}, a_{33}, b_{11}$, and b_{12} are incorrect, and a_{23} is omitted. Bracketed terms above highlight the errors. Hokari et al. have an extra α multiplying each bracketed term in a_{21} and a_{22}, a_{33} as a positive term, and they add the bracketed terms in b_{11} and b_{12}. The finished Excel sheet output shown in their Figure 6 is replicable using the corrected values above.

The authors then use Farmer (1999)'s method to solve the system of equations (A10):

$$A\begin{bmatrix}\hat{c}_t\\ \hat{k}_t\\ \hat{z}_t\end{bmatrix} = B\begin{bmatrix}\hat{c}_{t+1}\\ \hat{k}_{t+1}\\ \hat{z}_{t+1}\end{bmatrix} + C\begin{bmatrix}\varepsilon_{t+1}\\ E_t\left(\hat{c}_{t+1}\right) - \hat{c}_{t+1}\\ E_t\left(\hat{z}_{t+1}\right) - \hat{z}_{t+1}\end{bmatrix}$$

$$\begin{bmatrix}\hat{c}_t\\ \hat{k}_t\\ \hat{z}_t\end{bmatrix} = A^{-1}B\begin{bmatrix}\hat{c}_{t+1}\\ \hat{k}_{t+1}\\ \hat{z}_{t+1}\end{bmatrix} + A^{-1}C\begin{bmatrix}\varepsilon_{t+1}\\ E_t\left(\hat{c}_{t+1}\right) - \hat{c}_{t+1}\\ E_t\left(\hat{z}_{t+1}\right) - \hat{z}_{t+1}\end{bmatrix}$$

They define:

$$M_0 = A^{-1}B$$

The eigenvalues of $M_0(\lambda)$, and matrix Q of eigenvectors are given by:

$$\lambda_1 = \frac{m_{11} + m_{22} - \sqrt{\left(m_{11} + m_{22}\right)^2 - 4\left(m_{11}m_{22} - m_{12}m_{21}\right)}}{2}$$

$$\lambda_2 = \frac{m_{11} + m_{22} + \sqrt{\left(m_{11} + m_{22}\right)^2 - 4\left(m_{11}m_{22} - m_{12}m_{21}\right)}}{2}$$

$$\lambda_3 = m_{33}$$

(A11) $$Q = \begin{bmatrix} m_{12} & m_{12} & \frac{m_{12}m_{23} - m_{13}(m_{22} - \lambda_3)}{(m_{11} - \lambda_3)(m_{21} - \lambda_3) - m_{12}m_{21}} \\ \lambda_1 - m_{11} & \lambda_2 - m_{11} & \frac{m_{21}m_{13} - m_{23}(m_{11} - \lambda_3)}{(m_{11} - \lambda_3)(m_{21} - \lambda_3) - m_{12}m_{21}} \\ 0 & 0 & 1 \end{bmatrix} \equiv \begin{bmatrix} q_{11} & q_{12} & q_{13} \\ q_{21} & q_{22} & q_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

Equation (A11) allows a simplification to:

$$M_0 = Q \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} Q^{-1}.$$

This simplification allows:

$$\begin{bmatrix} \hat{c}_t \\ \hat{k}_t \\ \hat{z}_t \end{bmatrix} = M_0 \begin{bmatrix} \hat{c}_{t+1} \\ \hat{k}_{t+1} \\ \hat{z}_{t+1} \end{bmatrix} + A^{-1}C \begin{bmatrix} \varepsilon_{t+1} \\ E_t(\hat{c}_{t+1}) - \hat{c}_{t+1} \\ E_t(\hat{z}_{t+1}) - \hat{z}_{t+1} \end{bmatrix}.$$

The solution of the linearization is then (A12)-(A15):

(A12) $$\hat{c}_t = \frac{q_{12}}{q_{22}}\hat{k}_t + \left(q_{13} - \frac{q_{12}q_{23}}{q_{22}}\right)\hat{z}_t.$$

(A13) $$\hat{k}_{t+1} = \frac{a_{21}\hat{c}_t + a_{22}\hat{k}_t + a_{23}\hat{z}_t}{b_{22}}.$$

(A14) $$\hat{z}_{t+1} = \rho\hat{z}_t + \epsilon_{t+1}.$$

(A15) $$\hat{l}_t = \frac{-\tau\hat{c}_t + \alpha\tau\hat{k}_t + \tau\hat{z}_t}{\alpha\tau - \frac{\gamma\bar{l}}{\left(1-\bar{l}\right)^2}}.$$

Equations (A16) and (A17) track the equilibrium wage and rental rate:

(A16) $$w_t = (1-\alpha)z_t k_t^{\alpha} l_t^{-\alpha}$$

(A17) $$r_t = \alpha z_t k_t^{\alpha-1} l_t^{1-\alpha} - \delta$$

Figure 1 below shows the Excel output for the shock sequence $\epsilon_t, t = 1, \ldots, 30$ as presented in Figure 6 in Hokari et al. (2007). Their spreadsheet uses the correct values of the ***A*** and ***B*** matrices since the Excel spreadsheet from this paper using the approximate shock values $\epsilon_t$reported in Figure 6 of Hokari et al. yield the same values of model variables when compared to Figure 1:

Figure 1: Replication of Simulated Model Results from Hokari et al. (2007)

t	epsilon(t)	(z(t)-z)/z	(k(t)-k)/k	(c(t)-c)/c	(l(t)-l)/l	(y(t)-y)/y	r(t)-r	(w(t)-w)/w	(i(t)-i)/i
0	0	0	0	0	0	0	0	0	0
1	3.468E-05	0.00003468	0	-7.97031E-06	0.000459169	0.000340946	1.40669E-05	-0.000118182	0.001721319
2	0.0000576	9.15517E-05	4.3033E-05	-2.44439E-06	0.001166228	0.000883756	3.46841E-05	-0.000282211	0.004389719
3	-1.35E-05	7.61291E-05	0.0001517	4.80599E-05	0.000846042	0.000690955	2.22461E-05	-0.000154962	0.003234362
4	-5.89E-05	1.56304E-05	0.000228767	9.52678E-05	-3.72295E-05	6.69776E-05	-6.67421E-06	0.000104204	-4.49432E-05
5	2.807E-05	4.33722E-05	0.000221924	8.5935E-05	0.00033738	0.000342305	4.96632E-06	4.93521E-06	0.00135655
6	4.818E-05	9.06414E-05	0.00025029	8.73295E-05	0.000932955	0.000796269	2.25221E-05	-0.000136547	0.003600957
7	8.673E-06	9.74109E-05	0.000334056	0.000121973	0.000933174	0.000831079	2.05017E-05	-0.000101969	0.003636427
8	-0.000046	4.93653E-05	0.000416616	0.000168692	0.000208921	0.000327449	-3.67714E-06	0.000118512	0.000955516
9	-4.26E-05	5.7286E-06	0.000430088	0.000184543	-0.000383216	-0.000106657	-2.21397E-05	0.000276591	-0.001258697
10	5.345E-06	1.09533E-05	0.000387868	0.000165098	-0.000268976	-3.92935E-05	-1.76199E-05	0.000229696	-0.0008479
11	0.0001319	0.000142623	0.000356974	0.000121486	0.001507332	0.001266886	3.75303E-05	-0.00024007	0.005798291
12	-1.05E-05	0.000129128	0.000493007	0.000183373	0.001183457	0.001082665	2.43203E-05	-0.000100602	0.004640424
13	5.453E-05	0.000180946	0.000596693	0.000216271	0.001758869	0.001552811	3.94301E-05	-0.000205598	0.006840398
14	-2.64E-05	0.000150747	0.000752785	0.000290666	0.001192409	0.001196761	1.83104E-05	4.4827E-06	0.004781432
15	0.0001127	0.000260281	0.000853501	0.000309016	0.00253516	0.002235449	5.6979E-05	-0.000298754	0.009856754
16	-9.88E-05	0.000156015	0.001078583	0.000430247	0.000914417	0.001125099	1.92333E-06	0.000210638	0.003874059
17	-1.09E-05	0.000141839	0.00114847	0.000463706	0.000652125	0.000959247	-7.79476E-06	0.00030701	0.002919693
18	6.672E-05	0.00020558	0.00119275	0.000468192	0.001448808	0.001569121	1.55219E-05	0.000120411	0.005924588
19	-1.26E-05	0.000188663	0.001311046	0.000523201	0.001098556	0.001357978	1.94277E-06	0.000259353	0.004660503
20	1.242E-05	0.000197121	0.001394783	0.000557443	0.001121165	0.0014094	6.11797E-07	0.000288143	0.004779892
21	-1.61E-05	0.000176881	0.00147941	0.000598666	0.00076286	0.001178353	-1.23984E-05	0.000415296	0.003471696
22	-8.35E-06	0.000164817	0.001529217	0.000622963	0.000549962	0.001040871	-2.01169E-05	0.000490677	0.002694189
23	2.063E-05	0.000181986	0.001558342	0.000631603	0.000746194	0.001198625	-1.48143E-05	0.000452206	0.003441863
24	6.423E-05	0.000242394	0.00160543	0.000638068	0.001495749	0.001774667	6.9868E-06	0.000278871	0.00627125
25	0.0000127	0.000250004	0.001722075	0.000686727	0.001471999	0.001805278	3.4428E-06	0.000333171	0.006230464
26	5.263E-05	0.000297384	0.001834785	0.000724545	0.001979014	0.002228369	1.62332E-05	0.000249434	0.008177764
27	0.0001206	0.000411739	0.001993359	0.00076679	0.003323834	0.003292525	5.35379E-05	-3.02191E-05	0.013284776
28	0.0000643	0.000467392	0.002275645	0.000875987	0.003759393	0.003732697	6.00345E-05	-2.53184E-05	0.015034341
29	-2.31E-05	0.000434477	0.002594612	0.001021392	0.002983133	0.003288232	2.85952E-05	0.000305412	0.012256252
30	-4.33E-05	0.000382053	0.002836153	0.001137821	0.002031217	0.002681313	-6.33746E-06	0.000649584	0.008787642
31	9.447E-05	0.0004685	0.00298494	0.00118225	0.003016976	-0.022544699	2.02098E-05	0.000457859	-0.116412678

Minor differences between the model results on a few entries are due to the rounding of the shock ϵ_t, so their reported simulation results are correct, but the method details were misreported[2].

[2] It should be noted that the figure suggests shock sizes in the second column that are too small given $\sigma = 0.0072$. These shock sizes are consistent with using σ as the shock variance, not the standard deviation. I correct the shock sizes in the workbook's Economy Simulation sheet.

Appendix B: Real Business Cycle Lesson Plan

This lesson plan helps instructors guide students to simulate the real business cycle (RBC) model and helps them evaluate the model's ability to match business cycle facts. Optional extensions in Section E introduce students to impulse responses to one-time and permanent shocks, including temporary productivity shocks, permanent productivity shocks, and the effects of a one-time loss of physical capital from a natural disaster. This lesson will work best as a supplement to discussions of the RBC model where instructors want to develop students' understanding of the model and its contributions.

A. Materials Needed

- Microsoft Excel (No add-ins needed)
- The Real Business Cycle Model Workbook

B. Goals and Lesson

Students' goals are to simulate the calibrated RBC model to generate realistic economic data and observe how the model matches many business cycle characteristics without explicitly targeting them. The main characteristics cited throughout the RBC literature, as summarized by Rebelo (2005), are:

1. Investment is about 3 times more volatile than output.
2. Nondurable consumption is less volatile than output.
3. Hours worked and output are similar in volatility.
4. Almost all macroeconomic variables are strongly procyclical.
5. There is substantial persistence in macroeconomic variables.

Instructors should note that the simulation shows deviations of macroeconomic variables from their steady state, which are comparable to the business cycle facts calculated from the HP-filtered data. The focus of the data and simulations is on the cycle component of the macroeconomic variables.

Instructors need to introduce or review three tools to show how the simulation results match the business cycle facts before continuing with the simulation below:

I. Correlation coefficient: The correlation coefficient between macroeconomic variables and output allows students to demonstrate their procyclicality (fact 4).

II. First order autocorrelation: The first order autocorrelation of macroeconomic variables allows students to show that there is substantial persistence in macroeconomic variables (fact 5).

III. Relative standard deviation: The relative standard deviation allows students to compare the variation in the macroeconomic variables to the variation in output (facts 1-3).

The definition sheet below provides students with additional information about these calculations.

C. The Simulation

The simulation controls and results are in the "Economy Simulation Main Sheet." Students should choose a positive integer seed between 1 and 2,147,483,647 in cell B2 of the "Economy Simulation Main Sheet.[3]" This choice sets the random number generator for the shocks ϵ_t for all 1,000 periods of the simulation and is the only thing that the students need to select in the workbook. Once the seed is chosen, the simulation on the "Economy Simulation Calculations" sheet automatically updates the simulation outcomes, and the "Economy Simulation Main Sheet" shows graphs of the deviations of main economic variables from trend and simulation statistics.

D. Interpreting the Results

Students need to interpret the simulation statistics to complete the worksheet. The worksheet guides students to choose the statistical measure that describes the relationship between the variables in each fact, and the students should use the following measures for each fact:

1. Investment is about 3 times more volatile than output: examine the relative standard deviation between investment and output.
2. Nondurable consumption is less volatile than output: examine the relative standard deviation between consumption and output.
3. Hours worked and output are similar in volatility: examine the relative standard deviation between labor and output[4].
4. Almost all macroeconomic variables are strongly procyclical: examine the correlation between macroeconomic variables and output.
5. There is substantial persistence in macroeconomic variables: examine the first order autocorrelation of macroeconomic variables.

Not every simulation will exactly fit every business cycle fact. For example, many simulations have investment 4 or more times more volatile than output, and the volatility of hours (labor) is usually lower than output.

E. Extensions: Impulse Responses

Studying the impulse responses from one-time shocks or permanent changes in the economy is another common exercise with this model that helps researchers understand how macroeconomic variables change and interact over time, understand the progression of the business cycle, and forecast the effects of events. There are several types of shocks that could be modeled with the sheet:

- **Productivity Shock**: The "Impulse Response Main Results" sheet by default shows the results from a one-time 1% positive productivity shock in quarter 1 of the simulation, and the graphs on that sheet show the responses of macroeconomic variables to these shocks. A -1% shock (-0.01 in B2) shows the recessionary effects of a negative produc-

[3] Conditional formatting highlights B2 as green when a valid number is in the cell and red if the seed is invalid. If students enter a decimal for the seed, it is rounded down to the nearest integer.

[4] Note: "hours worked" here refers to total hours worked across the economy, not the average number of hours worked per week, which is a common source of confusion for students.

tivity shock.

- **Natural Disaster:** Suppose that a natural disaster destroyed some of the physical capital of the country. This disaster could be modeled as a one-time loss of capital in time $t = 1$. First, shut down the productivity shock by setting the productivity shock to 0% (0 in B2 in the "Impulse Response Main Results" sheet). Then, model the shock in the "Impulse Response Calculations" sheet by overriding the capital stock in time $t = 1$ to -0.01. The impulse responses on the "Impulse Responses Main Results" sheet update to show the economy's recovery to the steady state after the capital loss.

- **Permanent increase in technology:** Suppose there is a discovery that leads to a 1% increase in productivity in the economy. It can be modeled as a permanent, immediate increase to productivity or a gradual increase in productivity with slow adoption of the technology.

    - If the new technology immediately increases productivity, then the change could be modeled as a permanent increase in z_t. First, shut down the productivity shock by setting the productivity shock to 0% (=0 in B2 in the "Impulse Response Main Results" sheet). Then, force the productivity value z_t to be 0.01 for time $t = 1$ until the end of the simulation by replacing cells B31 through B1031 with =0.01 in the "Impulse Response Calculations" sheet.

    - If there is a gradual increase in innovation and adoption that eventually leads to a 1% increase in productivity, then the transition to the new steady state equilibrium can be modeled as a permanently higher productivity shock. First, shut down the productivity shock by setting the productivity shock to 0% (=0 in B2 in the "Impulse Response Main Results" sheet). Then, replace all shocks in column B of the "Impulse Response Calculations" sheet from time $t = 1$ to the last period of the simulation with $-q(1 - \rho)$ where q is the desired eventual steady state fraction increase in z_t in the economy. For example, to model an increase in productivity such that z_t is permanently 1% higher (z_t eventually becomes 0.01), replace cells B31 through B1031 with =0.01*(1-B7) in the "Impulse Response Calculations" sheet.

References

Rebelo, S. (2005). Real business cycle models: Past, present, and future. *Scandinavian Journal of Economics*, 107(2), 217-238. DOI: 10.1111/j.1467-9442.2005.00405.x

Appendix C: Real Business Cycle Definition Sheet

Correlation Coefficient:

(1)
$$\mathrm{corr}\left(\mathrm{X_t}, \mathrm{Y_t}\right) = \frac{\sum_{\mathrm{t=1}}^{\mathrm{T}}\left(\mathrm{X_t} - \overline{\mathrm{X}}\right)\left(\mathrm{Y_t} - \overline{\mathrm{Y}}\right)}{\sqrt{\sum_{\mathrm{t=1}}^{\mathrm{T}}\left(\mathrm{X_t} - \overline{\mathrm{X}}\right)^2 \sum_{\mathrm{t=1}}^{\mathrm{T}}\left(\mathrm{Y_t} - \overline{\mathrm{Y}}\right)^2}}$$

$\overline{\mathrm{X}}$*: sample mean of variable* $\mathrm{X_t}$ *from simulation*
$\overline{\mathrm{Y}}$*: sample mean of output* $\mathrm{Y_t}$ *from simulation*

The correlation coefficient is a measure between -1 and 1 that describes the strength and direction of the linear relationship between variables $\mathrm{X_t}$and $\mathrm{Y_t}$. A value of +1 indicates a perfect positive relationship: as one variable increases, the other also increases proportionally. A value of -1 indicates a perfect negative relationship: as one variable increases, the other decreases proportionally. A value of 0 indicates no linear relationship between the variables. In this exercise, the correlation coefficient of $\mathrm{X_t}$with output examines how closely variable $\mathrm{X_t}$ trends over time with output.

Hodrick-Prescott Filter (and Trend/Cycle Components):

The Hodrick-Prescott filter is a method to detrend time series macroeconomic variables that estimates the long-term trend of the variable and the cycle of the variable around the trend. Suppose $\mathrm{Y_t}$ is raw, seasonally adjusted real gross domestic product (output) data. Then, the filter estimates the long-term trend in output $\mathrm{Y_{trend,t}}$ and the cycle $\mathrm{Y_{cycle,t}}$ as:

(2)
$$\mathrm{Y_t} = \mathrm{Y_{trend,t}} + \mathrm{Y_{cycle,t}}$$

Rearranging, the cycle component is:

(3)
$$\mathrm{Y_{cycle,t}} = \mathrm{Y_t} - \mathrm{Y_{trend,t}}$$

In this worksheet, all real economic variables are expressed in terms of their cycle components so that positive values mean that the variables are above their long-term trend values, and negative values mean they are below their long-term trend values.

First Order Autocorrelation:

The first order autocorrelation (foc) examines how closely related consecutive values of a time series variable are, and it is computed as the correlation coefficient between a variable and its one-period-lagged values:

(4)
$$\mathrm{foc}(\mathrm{X_t}) = \mathrm{corr}(\mathrm{X_t}, \mathrm{X_{t-1}})$$

The first order autocorrelation can range between -1 and 1. Positive values closer to 1 indicate that a variable was positive in the previous period (above trend in our variables) is likely to remain positive in the current period. Negative values closer to -1 indicate that a variable that was positive in the previous period is like to become negative (below trend) in the current period. Macroeconomic variables are more persistent when the first order autocorrelation is positive and closer to 1 since macroeconomic variables that are above trend are likely to remain above trend in successive periods and vice versa.

Procyclical (and Countercyclical):

Procyclical economic variables are variables that move in the same direction as the economy typically as measured by output. A variable is procyclical if its cycle component is positively correlated with the cycle component of output. A variable is countercyclical if its cycle component is negatively correlated with the cycle component of output.

Relative Standard Deviation:

The relative standard deviation (rsd) of a variable is the ratio of the standard deviation of a variable to the standard deviation of a comparison variable:

(5)
$$\mathrm{rsd}\left(\mathrm{X_t}, \mathrm{Y_t}\right) = \frac{\mathrm{sd}(\mathrm{X_t})}{\mathrm{sd}(\mathrm{Y_t})}.$$

The comparison variable is output $(\mathrm{Y_t})$ for all relevant comparisons in this exercise. If variable $\mathrm{X_t}$ is consumption, then a relative standard deviation less than 1 indicates that consumption is less volatile than output, and a relative standard deviation greater than 1 indicates that consumption is more volatile than output. A relative standard deviation near 1 indicates consumption is similarly volatile as output.

Appendix D: Example Worksheet

Seed chosen: ____________________

Stylized Fact	Measure(s)	Value(s)	Fits Fact?
Investment 3 Times More Volatile than Output			
Nondurable Consumption Less Volatile than Output			
Hours Worked and Output Similarly Volatile			
Almost all Macroeconomic Variables Strongly Procyclical[1]			
Substantial Persistence of Macroeconomic Variables			

[5] Excluding the real interest rate.

Appendix E: Example Answer Key

Seed chosen: 2

Stylized Fact	Measure	Value(s)	Fits Fact? Why?
Investment 3 Times More Volatile than Output	Relative standard deviation between investment and output	3.27	Yes, investment is just over 3x as volatile as output
Nondurable Consumption Less Volatile than Output	Relative standard deviation between consumption and output	0.64	Yes, the relative standard deviation is less than 1
Hours Worked and Output Similarly Volatile	Relative standard deviation between output and labor	0.81	No, hours worked are less volatile than output
Almost all Macroeconomic Variables Strongly Procyclical[1]	Correlation with Output	Range from 0.59 to 0.97	Yes, most macroeconomic variables are strongly positively correlated with output
Substantial Persistence of Macroeconomic Variables	First order autocorrelation of simulated variables	Range from 0.88 to just under 1	Yes, the first order autocorrelations are positive and close to 1

[5] Excluding the real interest rate.