JOURNAL OF
ECONOMICS
TEACHING

Discussing Substitutes and Complements in an Economics Principles Classroom with Discrete Choice Consumer Utility Tables

This paper extends the strategy espoused by Holmgren (2017) to employ discrete choice utility tables to teach the consumer choice model in introductory microeconomics classes. These extensions show instructors how to explain the equal marginal utility per dollar spent rule using discrete choice tables. One can also examine the more basic issue of the meaning of marginal utility and the ordinal nature of utility using these tables. This paper also provides additional tables that allow instructors to distinguish between goods that are substitutes or complements and to show why an individual may choose to not purchase a good once its price increases sufficiently. Finally, this hyperlink provides an Excel file that allows instructors to create new tables as well as graphical overlays that instructors may use to bridge between tabular display and graphical image.

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1. Introduction

This paper provides a series of alternative discrete choice utility tables that allow instructors to extend the discussion of the consumer choice problem. Many instructors do not cover consumer choice, because introductory texts often relegate it to appendices or later chapters. Holmgren (2017) argues that some form of indifference analysis should be part of introductory microeconomics courses given that many students go on to second courses in microeconomics in which indifference curves play a starring role. A more substantive reason to cover this material is that students have already confronted consumer choice in everyday life. It therefore provides one of the easiest ways to teach students the marginal tradeoffs that are at the heart of microeconomic analysis. Additionally, indifference curves and budget constraints are topics that have ready analogs on the producer side both at the introductory and intermediate level. Although this paper primarily is targeted at introductory microeconomics courses, it may well be helpful to instructors teaching elective courses for which intermediate level theory is not required. It provides such instructors a path to build the necessary scaffolding to explain concepts such as the equal marginal-benefit-per-dollar-spent principle and to allow students to understand rudimentary indifference curve or isoquant models. These tables can be used in the classroom and as an assessment tool (for homework or exam). Rather than reprise multiple versions of each unique table, this paper will simply lay out the multiple version strategy for analysis that Holmgren (2017) suggested in Section 2.

Sections 3 through 5 examine extensions, alternative strategies, and additional topics for discussion using these tables. Finally, this paper provides additional discrete choice utility tables for classroom use and assessment, and graphical overlays that allow instructors to bridge these discrete utility tables to their graphical counterparts. An appendix discusses the interactive Excel file that produced these tables. This file allows the instructor to create lectures that build out various components of the analysis. One need not pursue all of these extensions in class, but instead you may choose among these extensions depending time constraints.

2. Holmgren's Strategy for using Discrete Choice Tables in the Classroom

Holmgren (2017) uses the economist's standby – the equal weighted Cobb-Douglas utility function, $U(x, y) = x \cdot y$, for the majority of his analysis. He modifies the resulting tables in one cell, (7, 14), in order to show how the income and substitution effect can be viewed in tabular form, even without formally using indifference curves.

Holmgren (2017) lays out a series of tables, all based on the same underlying preferences, that show students how to analyze the consumer choice problem in a discrete choice context. He uses these tables to introduce the concept of indifference curves (in the discrete case, the points are not curves but the set of indifferent bundles). Once these indifferent bundles are noted, one can conceptually connect between them to obtain indifference curves. Students also are introduced to the concept of affordable bundles and how the budget constraint changes for different prices of good x. Given budget constraints, it is an easy task to find the highest utility bundle among the affordable set of bundles. Next, he shows two compensated budget constraints, one for fixed-basket, and the other for fixed utility. These versions allow the instructor to discuss the substitution bias of a price increase, as well as the income and substitution effects of a price increase. The latter allows students to have a second approach to learning about income and substitution effects, and it allows the instructor to discuss normal versus inferior goods in a tabular context.

Although the vast majority of the analysis employs a single set of preferences, Holmgren (2017) provides a final table based on a different set of preferences. This allows students to see what preferences look like when x is an inferior good because the substitution bundle for the increased price of x, at (4, 17), has less x than the final bundle at (5, 10). This table is provided without building out various versions and it therefore offers students the chance to analyze the consumer choice problem on their own as a homework or exam question, even if students have read his article.

3. Simple Extensions for Classroom Analysis

Two principles are taught using utility analysis. One is the "law of diminishing marginal utility" and the other is the equal marginal-utility-per-dollar-spent rule (also known as the equal bang-for-the-buck rule) used for utility maximization. Because utility is ordinal, the first is a white lie introductory microeconomics instructors often tell to sell the basic point. The second is an immutable law that returns on the production side to provide the rule for finding the cost-minimizing input bundle. Both can be addressed in the present context.

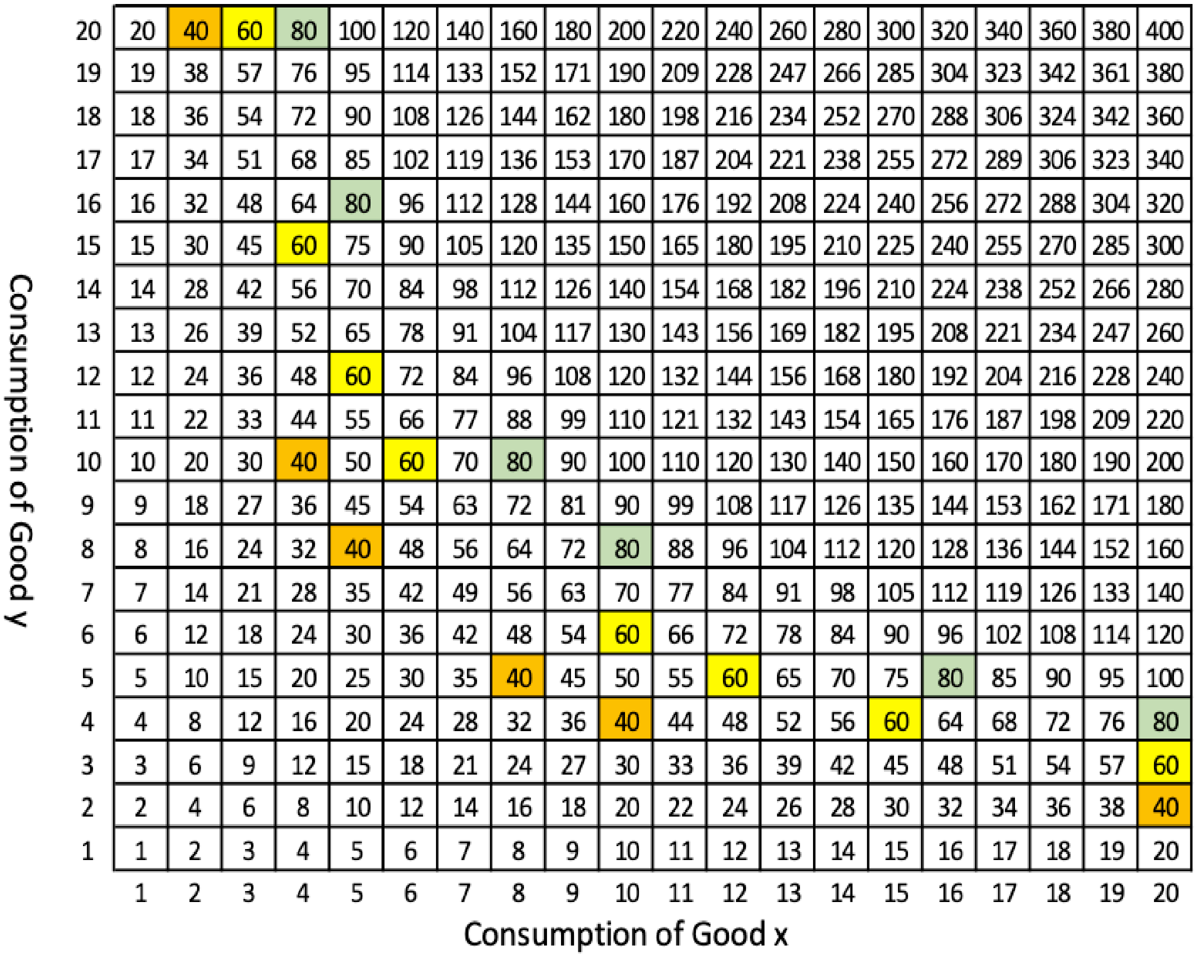
A. The Meaning of Utility and How Utility Represents Preferences

Students balk at putting utility level values on bundles of goods for good reason. We want students to focus on bundles that have equal utility, and we do not want them to focus in on the utility level, per se, because utility is an ordinal concept. Table 1 and Table 2 show the basic point.

Table 1 has the same preferences as Holmgren's (2017) Tables 2 through 6, $U(x, y) = x \cdot y$ (without bundle (7, 14) reset to $U(7, 14) = 100$). Three indifference curves are highlighted in the table. This table provides a platform to make a number of points.

Students often perceive a "problem" with Table 1. If the bundle (10, 10) provides 100 utils, then why should the bundle (20, 20) provide 400 utils, four times as much utility? This same point can be made starting from any (x, y) bundle as long as x and y both are less than or equal to 10 (so that the "double" bundle is represented on the table). The perceptive student may argue, "If I have twice as much, why shouldn't I be twice as happy?" When I do not get this response, I ask students to consider (10, 10) versus (20, 20) in order to elicit this response. This allows me to discuss two points: 1) the ordinal nature of utility, and 2) the common mistake of conflating preferences with affordability. One talks about being happier, but not twice as happy, and preferences (and their numerical representation via a utility function) are independent of budget and price. These points lead to a discussion of what it means to have a utility function that represents preferences. Consider the utility function, $V(x, y) = 10 \cdot (x \cdot y)^{0.5}$, shown in Table 2.

Table 1 – Three Indifference Curves: Six Bundles with U = 40, Eight Bundles with U = 60, and Six Bundles with U = 80, Given U = x·y

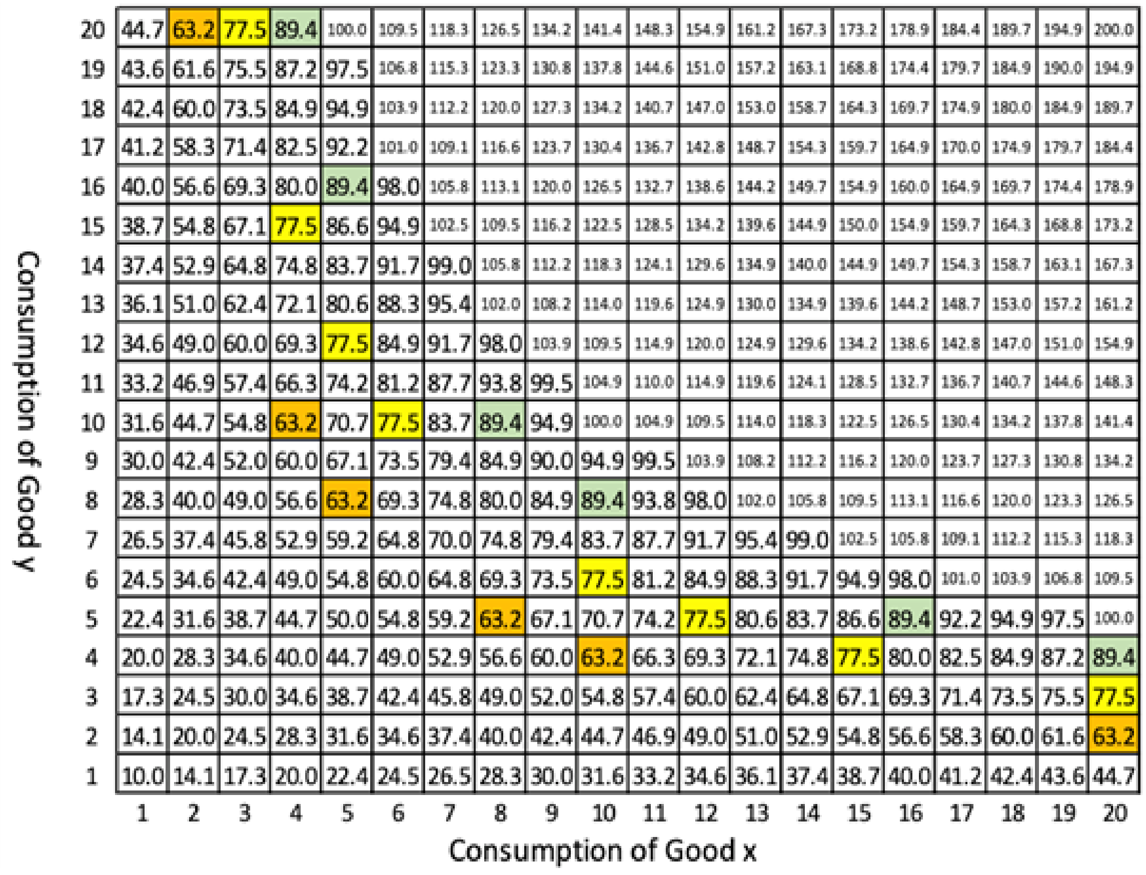


20	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400
19	19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361	380
18	18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342	360
17	17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323	340
16	16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320
15	15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300
14	14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266	280
13	13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247	260
12	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240
11	11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209	220
10	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200
9	9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180
8	8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160
7	7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140
6	6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120
5	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
4	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80
3	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60
2	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40
1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

Consumption of Good y

Consumption of Good x

Table 2 – The Same Bundles Have V = 63.2, V = 77.5, and V = 89.4, Given $V = 10\cdot(x\cdot y)^{0.5}$



Consumption of Good y

20	44.7	63.2	77.5	89.4	100.0	109.5	118.3	126.5	134.2	141.4	148.3	154.9	161.2	167.3	173.2	178.9	184.4	189.7	194.9	200.0
19	43.6	61.6	75.5	87.2	97.5	106.8	115.3	123.3	130.8	137.8	144.6	151.0	157.2	163.1	168.8	174.4	179.7	184.9	190.0	194.9
18	42.4	60.0	73.5	84.9	94.9	103.9	112.2	120.0	127.3	134.2	140.7	147.0	153.0	158.7	164.3	169.7	174.9	180.0	184.9	189.7
17	41.2	58.3	71.4	82.5	92.2	101.0	109.1	116.6	123.7	130.4	136.7	142.8	148.7	154.3	159.7	164.9	170.0	174.9	179.7	184.4
16	40.0	56.6	69.3	80.0	89.4	98.0	105.8	113.1	120.0	126.5	132.7	138.6	144.2	149.7	154.9	160.0	164.9	169.7	174.4	178.9
15	38.7	54.8	67.1	77.5	86.6	94.9	102.5	109.5	116.2	122.5	128.5	134.2	139.6	144.9	150.0	154.9	159.7	164.3	168.8	173.2
14	37.4	52.9	64.8	74.8	83.7	91.7	99.0	105.8	112.2	118.3	124.1	129.6	134.9	140.0	144.9	149.7	154.3	158.7	163.1	167.3
13	36.1	51.0	62.4	72.1	80.6	88.3	95.4	102.0	108.2	114.0	119.6	124.9	130.0	134.9	139.6	144.2	148.7	153.0	157.2	161.2
12	34.6	49.0	60.0	69.3	77.5	84.9	91.7	98.0	103.9	109.5	114.9	120.0	124.9	129.6	134.2	138.6	142.8	147.0	151.0	154.9
11	33.2	46.9	57.4	66.3	74.2	81.2	87.7	93.8	99.5	104.9	110.0	114.9	119.6	124.1	128.5	132.7	136.7	140.7	144.6	148.3
10	31.6	44.7	54.8	63.2	70.7	77.5	83.7	89.4	94.9	100.0	104.9	109.5	114.0	118.3	122.5	126.5	130.4	134.2	137.8	141.4
9	30.0	42.4	52.0	60.0	67.1	73.5	79.4	84.9	90.0	94.9	99.5	103.9	108.2	112.2	116.2	120.0	123.7	127.3	130.8	134.2
8	28.3	40.0	49.0	56.6	63.2	69.3	74.8	80.0	84.9	89.4	93.8	98.0	102.0	105.8	109.5	113.1	116.6	120.0	123.3	126.5
7	26.5	37.4	45.8	52.9	59.2	64.8	70.0	74.8	79.4	83.7	87.7	91.7	95.4	99.0	102.5	105.8	109.1	112.2	115.3	118.3
6	24.5	34.6	42.4	49.0	54.8	60.0	64.8	69.3	73.5	77.5	81.2	84.9	88.3	91.7	94.9	98.0	101.0	103.9	106.8	109.5
5	22.4	31.6	38.7	44.7	50.0	54.8	59.2	63.2	67.1	70.7	74.2	77.5	80.6	83.7	86.6	89.4	92.2	94.9	97.5	100.0
4	20.0	28.3	34.6	40.0	44.7	49.0	52.9	56.6	60.0	63.2	66.3	69.3	72.1	74.8	77.5	80.0	82.5	84.9	87.2	89.4
3	17.3	24.5	30.0	34.6	38.7	42.4	45.8	49.0	52.0	54.8	57.4	60.0	62.4	64.8	67.1	69.3	71.4	73.5	75.5	77.5
2	14.1	20.0	24.5	28.3	31.6	34.6	37.4	40.0	42.4	44.7	46.9	49.0	51.0	52.9	54.8	56.6	58.3	60.0	61.6	63.2
1	10.0	14.1	17.3	20.0	22.4	24.5	26.5	28.3	30.0	31.6	33.2	34.6	36.1	37.4	38.7	40.0	41.2	42.4	43.6	44.7
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

Consumption of Good x

The numbers in Table 2 are shown to the nearest 0.1 because many bundles now have non-integer values associated with that bundle. The number associated with any given bundle has changed (except for three bundles: (5, 20); (10, 10); and (20, 5)), but the set of bundles indifferent to any given bundle has not changed.[1] Note, in particular, that the indifference curve associated with V = 63.2 in Table 2 is the same as U = 40 in Table 1. The same is true for V = 77.5 and U = 60, and V = 89.4 and U = 80. The takeaway from all of this: Both U and V represent the same underlying preferences.

To reinforce this point, you could introduce Figure 1, which shows continuous indifference curves through five (x, y) bundles where x = y: (4, 4), (5, 5), (6, 6), (7, 7), and (8, 8). Students can readily fill in the table because even those with weak math skills can recall perfect squares.[2]

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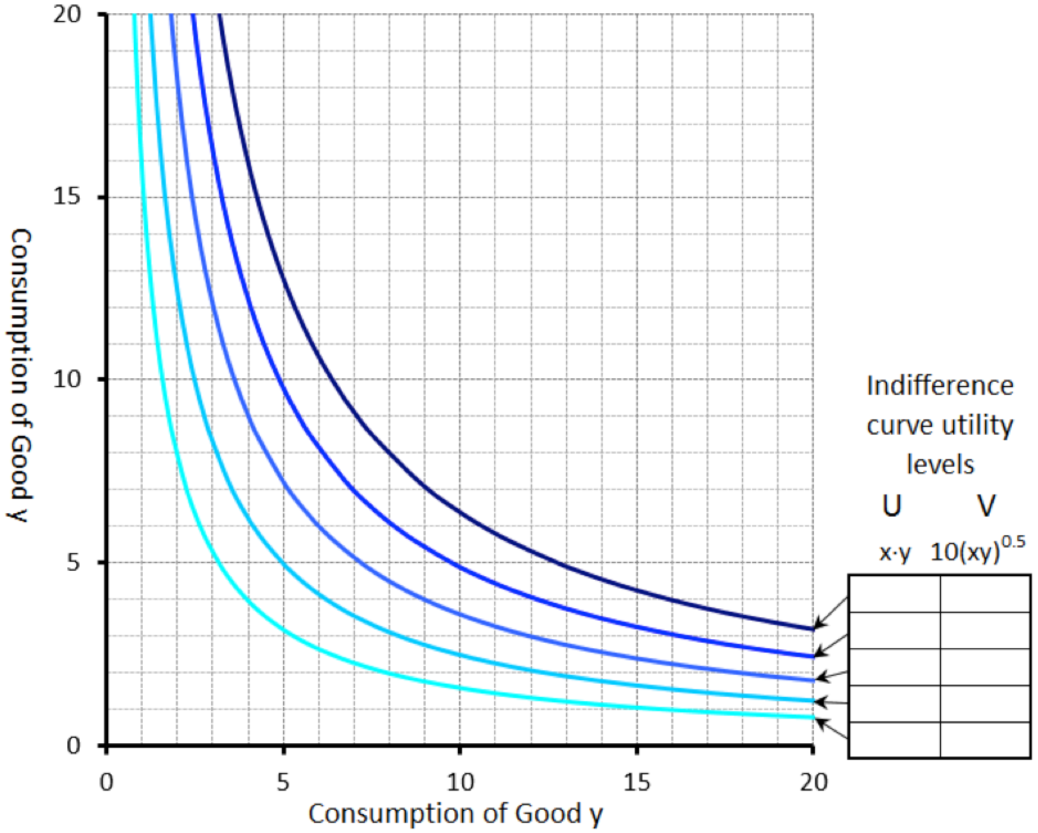
[1] This follows because V is a monotonic transformation of U; $V = 10\cdot U^{0.5}$.

[2] Values of U from lowest to highest are 16, 25, 36, 49 and 64 and values of V are 40, 50, 60, 70, and 80. These sets of values can be found along the x = y diagonal in Tables 1 and 2.

Figure 1 – A Continuous Indifference Map Based on Table 1 [with $U = x{\cdot}y$] and Table 2 [with $V = 10{\cdot}(x{\cdot}y)^{0.5}$], through $x = y = n$ Bundles for $n = 4, 5, 6, 7$, and 8



The indifference curves do not depend which utility function you use to represent preferences. Different U or V values lead to different indifference curves, but only bundles with the same utility level will remain on the same indifference curve. To further tie the tables and figure together you might ask which indifference curves in Tables 1 and 2 sandwich the dark blue outer indifference curve through (8, 8) in Figure 1. When asking this, you may want to point out to students that cells in Tables 1 and 2 are grid points in Figure 1. Using this, it is easy to see that yellow is below and green is above the dark blue indifference curve. The instructor can point out the eight U = 60 yellow bundles in Table 1 that are just below the dark blue U = 64 indifference curve in Figure 1. The same could be said had we used V instead (the eight V = 77.5 bundles in Table 2 are just below dark blue V = 80 indifference curve in Figure 1). U and V provide the same information.

B. Marginal Utility and the Law of Diminishing Marginal Utility

Marginal utility (MU) is the additional utility you obtain from one more unit of the good. It is worth noting that this depends not only on how much of the good you already have, but on how much of the other good you already have as well. In this two-good model, MU_x is the increment to utility you obtain from one more unit of MU_x and MU_y is the increment in utility you obtain from one more unit of y.

In Table 1, MU is constant. Point out that MU_x is equal to 20 for the top row (y = 20) but equal to 1 for the bottom row (y = 1). Similarly, MU_y is equal to 20 for the far right column (x = 20) but equal to 1 for the far left column (x = 1). More generally, $MU_x = y$ and $MU_y = x$, given the utility function depicted in Table 1. This, of course, flies in the face of the law of diminishing marginal utility, which is often a centerpiece of introductory treatments of the consumer choice model.

By contrast, the utility function shown in Table 2 satisfies the law of diminishing marginal utility. Subtracting subsequent V values along a row or column produces smaller and smaller increments in utility given a fixed amount of the other good. This is perhaps easiest to see along the bottom row (y = 1) or the first column (x = 1) where the second unit of the good has MU = 4.1 (4.1 = 14.1–10) and the 20th unit has MU = 1.1 (1.1 = 44.7–43.6).[3]

An aside on cardinality: The V values shown in Table 2 satisfy the mistaken notion that twice as much x and y should give twice as much utility. For example, consider V utility levels $V(8, 2) = 40 = 2\cdot V(4, 1) = 2\cdot 20$. If students persist in asking about this, it is worthwhile to point out that there is a strong rationale for this concept on the production side due to the cardinality of production. (If you are producing chairs, producing 40 chairs is twice as many as 20 chairs and it is 20 more chairs. Both notions (twice as many, 20 more) have cardinal meaning. It does not merely mean more chairs.)

An aside on MRS: One can take successive bundles on the U = 60 or V = 77.5 indifference curve in Tables 1 and 2 to introduce the concept of the marginal rate of substitution (MRS). MRS is the amount of y the individual is willing to give up to get one more unit of x.[4] Given convex preferences, MRS declines as x increases along the indifference curve. Instructors can work through MRS of the fourth unit of x along this curve as going from MRS = 5 (between (3, 20) and (4, 15)), to 3, to 2, to 1 to ½ to ⅓ to ⅕. The first three MRS calculations are for increments in x of 1, while the last four require increments in x of larger than 1. These calculations are based on discrete x and y choices. I find it worthwhile to point out that MRS can also be described at a point by the equation $MRS = MU_x/MU_y$, an equation that is independent of specific utility function chosen to represent preferences because taking the ratio of marginal utilities removes the ordinal nature that is inherent in the individual marginal utility values.[5]

The final take away from Tables 1 and 2 is to tell students to not become fixated on actual utility values in a utility table, but instead focus attention on the bundles that have the same numerical utility value, and hence are on an indifference curve. Similarly, introductory students should not concern themselves with the underlying functions that produce any given utility table. What matters here are the numbers in the cells, and how one maximizes utility subject to the budget constraint.

C. The Consumer Choice Criterion: Spend All Income and Have $MU_x/P_x = MU_y/P_y$

The consumer choice criterion: spend all income (I) and choose the consumption bundle where marginal utility per dollar spent is equal across goods, can be seen using discrete choice utility tables. It is worth presenting this analysis without focusing on the underlying equation of the utility function. Table 3 is the same as Table 1, without the highlighted bundles but with the x = 0 column and y = 0 row added to the table. This inclusion makes discussing budget constraints easier because an individual can afford I/P_x units of x if no y is purchased or I/P_y units of y if no x is purchased. The consumer is on the budget constraint because each good has positive MU.

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[3] More generally, $MU_x(x_0, y_0) = 5(y_0/x_0)^{0.5}$ and $MU_y(x_0, y_0) = 5(x_0/y_0)^{0.5}$ at the point (x_0, y_0) given the utility function depicted in Table 2, although this need not, of course, be pointed out in an introductory classroom setting. Note that MUx is a declining function of x for fixed value of y and MU_y is a declining function of y for fixed value of x given these marginal utility functions.

[4] Most texts formally define this with subscripts as MRS_{yx}.

[5] See Erfle (2016), Appendix 4A for proof of both assertions.

The consumer wishes to maximize utility subject to a budget constraint. It is useful to have each parameter be distinct from others within the model. In this context, it helps to have the prices of x and y differ from one another. Because we wish to focus on what happens when the price of x changes, we will let y be numeraire by making P_y = \$1, so that the amount of y one could purchase if no x is purchased is simply y = I; put another way, (0, I) is on the budget constraint. If an individual is spending all of their income but wish to purchase one more unit of x then they must purchase less y. The amount of y that is forgone to get one more x depends on the price of x. When P_x = \$2, two units of y must be forgone to get one more unit of x, because one needs to free up \$2 to reallocate towards their purchases of x. In geometric terms, the slope of the budget constraint is -2. By contrast, if P_x = \$0.50, then purchasing one less unit of y frees up \$1, which allows the individual to purchase two more units of x. The budget constraint in this instance has slope of -0.5.

Table 3 – A Discrete Choice Utility Table Where x and y Are Independent Goods

20		20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400
19		19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361	380
18		18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342	360
17		17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323	340
16		16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320
15		15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300
14		14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266	280
13		13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247	260
12		12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240
11		11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209	220
10		10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200
9		9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180
8		8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160
7		7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140
6		6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120
5		5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
4		4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80
3		3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60
2		2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40
1		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0																					
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

Consumption of Good y

Consumption of Good x

What affordable bundle has the highest utility given I = \$12, P_x = \$0.50 and P_y = \$1 in Table 3? If no x is purchased, 12 units of y could be purchased and if no y is purchased, 24 units of x could be purchased. However, the highest x value shown, 20 units, costs the individual \$10 leaving \$2 to spend on y. Put another way, (20, 2) is affordable and on the budget constraint. Between these two bounds, are other bundles that are also just affordable such as (2, 11), (4, 10), and so on. Each unit of y forgone allows two more units of x to be purchased. The highest utility bundle on this line is the bundle (12, 6) where U(12, 6) = 72. One can search among affordable bundles to find this solution but a more parsimonious strategy is to compare the marginal utility per dollar spent at various bundles on the budget constraint to find the bundle where marginal utility per dollar spent is equal across goods. It is worth noting that a dollar spent on a good generates 1/P units of that good (if price is \$2, a dollar purchases a half unit, but if price is \$0.50 then a dollar purchases two units of the good) so having equal marginal utility per dollar spent can be restated as having $MU_x/P_x = MU_y/P_y$. At (2, 11) for example, $MU_x/P_x = 11/\$0.50 > 2/\$1 = MU_y/P_y$. so that utility will increase if more x is purchased (along with less y). The same inequality holds for other bundles on the budget constraint as long as $x < 12$. Conversely at (18, 3), $MU_x/P_x = 3/\$0.50 < 18/\$1 = MU_y/P_y$. and utility increases if less x is purchased (along with more y). This inequality remains as long as $x > 12$. At (12, 6), the consumer has maximized their utility subject to the budget constraint by choosing a point where $MU_x/P_x = 6/\$0.50 = 12/\$1 = MU_y/P_y$.

This same strategy works regardless of price and income level. Had the price of x been \$2 with income of \$12 and Py = \$1, the consumer would choose (3, 6) and have U(3, 6) = 18 because (3, 6) is on the budget constraint, \$12 = \$2·3 + \$1·6, and $MU_x/P_x = 6/\$2 = 3/\$1 = MU_y/P_y$. And, had the price changed from \$0.50 to \$2 then the consumer would move from (12, 6) to (3, 6).

4. Showing the Substitution and Income Effect of a Price Change Using Utility Tables

Consumption changes when price changes for two reasons: the consumer substitutes towards the good that is becoming less expensive on a relative basis due to the price change and the price change alters the individual's real purchasing power. These are the substitution and income effects discussed in every introductory microeconomics text. These effects can be seen in the discrete choice consumer utility tables. Holmgren's approach, noted above, is to alter the utility value in (7, 14) from 98 to 100 in order to accomplish this discussion given that the price of x increases from \$1 to \$2.[6] While most students may not notice, Holmgren's (7, 14) substitution bundle no longer satisfies the equal MU/P rule at the new price.[7] A more parsimonious solution is to choose a price change where one obtains integer solutions for all three bundles (initial, final, and substitution). In this event, the substitution bundle will satisfy the equal MU/P rule.

A. The General Strategy to Find the Substitution Bundle

The individual substitutes towards the good that is now less expensive even if its price has not changed. If the price of x increases, then the price of y is becoming less expensive on a relative basis, even though the price of y has not changed. The individual will benefit from consuming a more y-intensive bundle in this instance. The reverse holds true if the price of x declines.

Take the increase in the price of x from \$0.50 to \$2 discussed above given I = \$12 and $P_y = \$1$. The *total effect* of the price increase is the move from (12, 6) to (3, 6), given the preferences shown in Table 3. Suppose we wish to consider the least costly way to achieve the initial utility level (of 72) after the price increase. Six bundles in Table 3 achieve this utility level but only one, (6, 12), does so at lowest cost given the new price of x. This is known as the s*ubstitution bundle*. This bundle maintains utility at the initial level and has equal marginal utility per dollar spent

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[6] Similarly, by changing *U*(7, 7) = 50 (rather than 49 in Table 3), we could discuss the income and substitution effect of a price decrease. Finally, by changing *U*(14, 7) = 100 (rather than 98 in Table 3), we could flip the analysis and discuss the income and substitution effect of an increase in the price of y from \$1 to \$2. The Excel file provides this altered table in rows 2-25 of the "Utility Tables" sheet.

[7] Given this change, $MU_x = 12$ and $MU_y = 5$ at (7, 14) so that $MU_x/2 > MU_y/\$1$.

using new prices ($MU_x/P_x = 12/\$2 = 6/\$1 = MU_y/P_y$). The move from the initial bundle to the substitution bundle is called the *substitution effect* of the price increase. Of course, an increase in price means that the *consumer* has less purchasing power with which to buy all goods. The move from the substitution bundle to the final bundle represents this change in purchasing power and is called the *income effect* of the price increase.

If the price of x had decreased instead from $2 to $0.50, then the total effect of the price decrease is the move from (3, 6) to (12, 6) given the preferences shown in Table 3. What is the least costly way to achieve the initial utility level (of 18) in the face of the price decrease? Six bundles in Table 3 achieve this utility level but only one, (6, 3), does so at lowest cost, given the new price of x. This is the substitution bundle because it maintains equal marginal utility per dollar spent using new prices ($MU_x/P_x = 3/\$0.50 = 6/\$1 = MU_y/P_y$). The move from the initial bundle to the substitution bundle is the substitution effect of the price decrease. A decrease in price means that the consumer has more purchasing power with which to buy all goods. The move from the substitution bundle to the final bundle is the income effect of the price decrease.

An aside on tangency of indifference curve and budget constraint: The general way to describe the decomposition of the total effect of a price change is to say that the individual moves along their initial indifference curve to the new price ratio and then jumps utility levels based on changes in purchasing power. The Excel file allows you to hide or show arrows for both of these effects. The substitution point has a MRS equal to the new price ratio. We can readily see this assertion from the equal marginal utility per dollar spent condition using the new prices by cross-multiplying: If $MU_x/P_x = MU_y/P_y$ then $MU_x/MU_y = P_x/P_y$, but above we noted that the slope of the indifference curve is the ratio of marginal utilities, $MRS = MU_x/MU_y$, and the slope of the budget constraint is P_x/P_y. Further, we can visually see this assertion by noting that the indifference curve overlay in the Excel file shows a tangency between the indifference curve and substitution budget constraint at the substitution bundle. The tangency condition is simply a restatement of the equal-marginal-utility-per-dollar-spent condition.

B. Required Compensation

Holmgren (2017) used the substitution bundle for the price increase to show that x is a normal good because the income effect on of a price increase in x is negative. It is worth pointing out that y is normal in this instance as well. The income effect shows more of both goods for the price decrease but less of both goods for the price increase. Holmgren also began foreshadowing a much deeper discussion of required compensation by asking how much compensation is required in order to maintain utility in the face of the price increase. This is, of course, the notion of compensating variation, which is beyond the scope of an introductory course. Nonetheless, it is a question worth asking even without building out the concept in detail in an introductory course. Such questions are especially useful for classes that focus attention on the public policy implications of economic changes.

For the case of a price increase discussed above, the compensation required to maintain utility in the face of the price increase is given by how much it would cost to purchase the substitution bundle (6, 12) given the new, higher price of x, $2. The bundle (6, 12) would cost $24 = $2·6 + 12 (you can show this substitution budget constraint using the graphic overlay). Because the individual already has $12 in income, they would require an additional $12 in compensation.

Had we instead examined the price decrease from $2 to $0.50, we could have taken $6 away from the individual and they would be just as happy as they were with the higher price.

It is worth working through the details of this assertion, because students do not as readily grasp taking away income as they do providing extra income in the face of price changes. The substitution bundle (6, 3) costs \$6 = \$0.5·6 + 3, which is \$6 more than the \$12 in income they originally had. Money must be taken away in order to maintain utility in the face of a price decrease.

C. Alternative Pricing Scenarios for Using Table 3

Twenty alternative scenarios (income, low price of x, high price of x) are shown in the "Price & Income Scenarios" sheet of the Excel file. Four of these scenarios involve different income levels for the prices of x discussed above (\$0.50 and \$2). The others have different high and low prices for x. Each produces exact integer results for all bundles. Ten of the scenarios have all four bundles with x < 20 and y < 20 so that interior solutions for both price increases and price decreases are readily discussed. Six more have one bundle on the boundary (x = 20 or y = 20) and one additional scenario has two bundles on the boundary. The remaining three are situations where the price-increase substitution bundle extends beyond the table because y > 20 so that you should only assign the price-decrease case. These scenarios provide significant flexibility in creating new questions from the table by simply providing the table with price and income assumptions. Answer keys are readily obtained, but more importantly, one can explain the answers using the graphical overlays when you go over the solutions in class.

Although we typically teach students to examine the effect of a change in the price of x, we can also ask that students examine a change in the price of y. Given the symmetry of Table 3, it is unsurprising (to us, but perhaps not to our students) that the results are mirror images of what we saw when we examined the change in the price of x. Nonetheless, it provides instructors the opportunity to test students' understanding of analyzing price changes. In Table 3, when P_y = \$0.50, the utility maximizing bundle is (6, 12), given I = \$12 and P_x = \$1. When P_y = \$2, the utility maximizing bundle is (6, 3), given I = \$12 and P_x = \$1. The substitution bundle involved in an increase in the price of y from \$0.50 to \$2 is (12, 6) and the substitution bundle for the decrease in the price of y from \$2 to \$1 is (3, 6) in this instance.

5. Utilizing the Cross-Price View to Distinguish between Substitutes and Complements

These tables can also be used to clarify three relations between two goods that are defined in introductory texts: substitutes, complements, and independent goods. We often resort to specific product choices to solidify these concepts. Coffee and tea are substitutes, or peanut butter and jelly are complements. But, instructors can use these tables to show how we can visually, and graphically, see the distinction between the three types of goods.

A. An Example Where x and y Are Independent of One Another

Take the increase n the price of x discussed in Section 4 using Table 3 above. The initial bundle is (12, 6), the final bundle is (3, 6), and the substitution bundle is (6, 12). Both goods are *normal* because the loss in income leads to lower demand for a normal good. An alternative reading of the same information suggests that x and y are *independent* because a change in the price of x does not change the consumption of y. By contrast, if an increase in price of x led to an increase in consumption of y, then y is a *substitute* for x. And, if an increase in price of x led to a decrease in consumption of y, then y is a *complement* of x.

One can use the "cross-price" view of substitution and income effects to see that this individual has preferences for which x and y are independent. When the price of x changes, consider what happens in the market for y (and vice versa if the price of y changed instead). In Table 3, the cross-price substitution effect, +6 units of y from (12, 6) to (6, 12), is the same magnitude as (but the opposite sign of) the cross-price income effect, -6 units of y from (6, 12) to (3, 6). The net effect is no change in y and the goods are independent. The Excel Screenshot in the appendix shows the graphical overlay depicting the income and substitution effect for this scenario.

B. Two Examples Where x and y Are Substitutes

When the two cross-price effects do not cancel one another, we end up with substitutes or complements. The next four tables examine two sets of preferences, both of which show that x and y are substitutes. Consider an increase in the price of x from \$1 to \$2. in Table 4. How would the consumer respond, given P_y = \$1 and I = \$20?

Table 4 – An Indifference Map Where *y* Is a Substitute for *x*

Consumption of Good y

20	128	148	168	187	206	224	242	259	276	292	308	324	340	355	370	385	400	414	428	442	456
19	122	142	162	180	198	216	233	250	266	282	298	313	329	344	358	373	387	401	415	429	442
18	117	136	155	173	191	208	225	241	257	272	288	303	317	332	346	360	374	388	401	415	428
17	111	130	148	166	183	200	216	232	247	262	277	292	306	320	334	348	361	375	388	401	413
16	105	124	142	159	175	191	207	223	238	252	267	281	295	308	322	335	348	361	374	386	399
15	100	118	135	151	167	183	198	213	228	242	256	270	283	296	309	322	335	347	360	372	384
14	94	111	128	144	159	175	189	204	218	231	245	258	271	284	297	309	321	334	345	357	369
13	88	105	121	136	151	166	180	194	208	221	234	247	259	272	284	296	308	320	331	342	354
12	82	98	114	129	143	157	171	184	197	210	223	235	247	259	271	283	294	305	316	327	338
11	76	91	106	121	135	148	161	174	187	199	211	223	235	246	258	269	280	291	301	312	323
10	70	85	99	113	126	139	152	164	176	188	200	211	222	234	244	255	266	276	286	297	307
9	64	78	91	105	118	130	142	154	166	177	188	199	210	220	231	241	251	261	271	281	290
8	57	71	84	96	109	121	132	144	155	165	176	187	197	207	217	227	236	246	255	265	274
7	51	64	76	88	100	111	122	133	143	154	164	174	184	193	203	212	221	230	239	248	257
6	44	56	68	79	91	101	112	122	132	142	151	161	170	179	188	197	206	214	223	231	240
5	37	49	60	71	81	91	101	111	120	130	139	148	156	165	173	182	190	198	206	214	222
4	30	41	52	62	72	81	90	99	108	117	126	134	142	150	158	166	174	182	189	197	204
3	23	33	43	52	62	71	79	88	96	104	112	120	128	135	143	150	157	164	171	178	185
2	16	25	34	43	51	60	68	76	83	91	98	106	113	120	127	133	140	147	153	160	166
1	8	17	25	33	41	49	56	63	70	77	84	91	97	104	110	116	123	129	135	141	146
0	0	8	16	23	30	37	44	50	57	63	69	75	81	87	93	99	104	110	115	121	126
y/x	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

Consumption of Good x

With I = \$20 and $P_x = P_y$ = \$1, the consumer maximizes utility by choosing the bundle (10, 10) in Table 4. If the price of x increases to \$2, the individual moves to (3, 14). The substitution bundle in this instance is (5, 17) because U(5, 17) = 200 and MU_x/P_x = 16/\$2 = 8/\$1 = MU_y/P_y. Because the cross-price substitution effect of the change in the price of x (+7 units of y) is larger than the cross-price income effect of the change in the price of x (-3 units of y), the net effect of an increase in price of x is an increase in demand for y of +4 units. This is a numerical example that exactly highlights the textbook definition of y being a substitute for x. Geometrically, we see that y is a substitute for x if the cross-price substitution effect dominates the cross-price income effect.

Instructors could then pose the required compensation question. In this instance, the answer is \$7 because (5, 17) costs \$27 given P_x = \$2. Since the individual already has \$20, \$7 extra is required to maintain initial utility after this price increase.

Instructors may also drive this home by asking, "How much less money would this individual need if they faced a price of \$1 rather than \$2 for x, given that P_y = \$1 and I = \$20? Students would solve this question by finding the substitution bundle for the decrease in the price of x. The answer is \$5 less income, but this requires students to apply what they have learned to a new situation. If needed, the instructor could prompt students with scaffolding questions such as: "What is the initial bundle and initial utility level? What is the final bundle and final utility level? What is the substitution bundle?" The answers are: (3, 14), U_0 = 144; (10, 10), U_1 = 200; and the substitution bundle is (7, 8) which costs \$15 given that the prices of both x and y are \$1. Note that, just as with the price increase, the cross-price substitution effect dominates the cross-price income effect because y is a substitute for x, regardless of whether the price of x increases or decreases.

One might follow this up by showing a graphical overlay of indifference curves and budget constraints highlighting the tangency condition that is equivalent to the equal-bang-for-the-buck rule. Table 5 shows the completed graph including the substitution and income effects of an increase in the price of x from \$1 to \$2.[8]

When examining goods that are substitutes for one another, one can imagine moving from purchasing some of each good to spending all income on only one of the two substitutes, especially if the increase in the price is substantial. Table 6 provides an example of such a boundary solution.

It is worth pointing out to students that the individual depicted in Table 6 appears to like y more than x because values above the y = x diagonal have higher values than their mirror counterparts below the diagonal. With P_x = \$1, the individual maximizes utility at (7, 13) where 35 percent of income is spent on x and 65 percent is spent on y.

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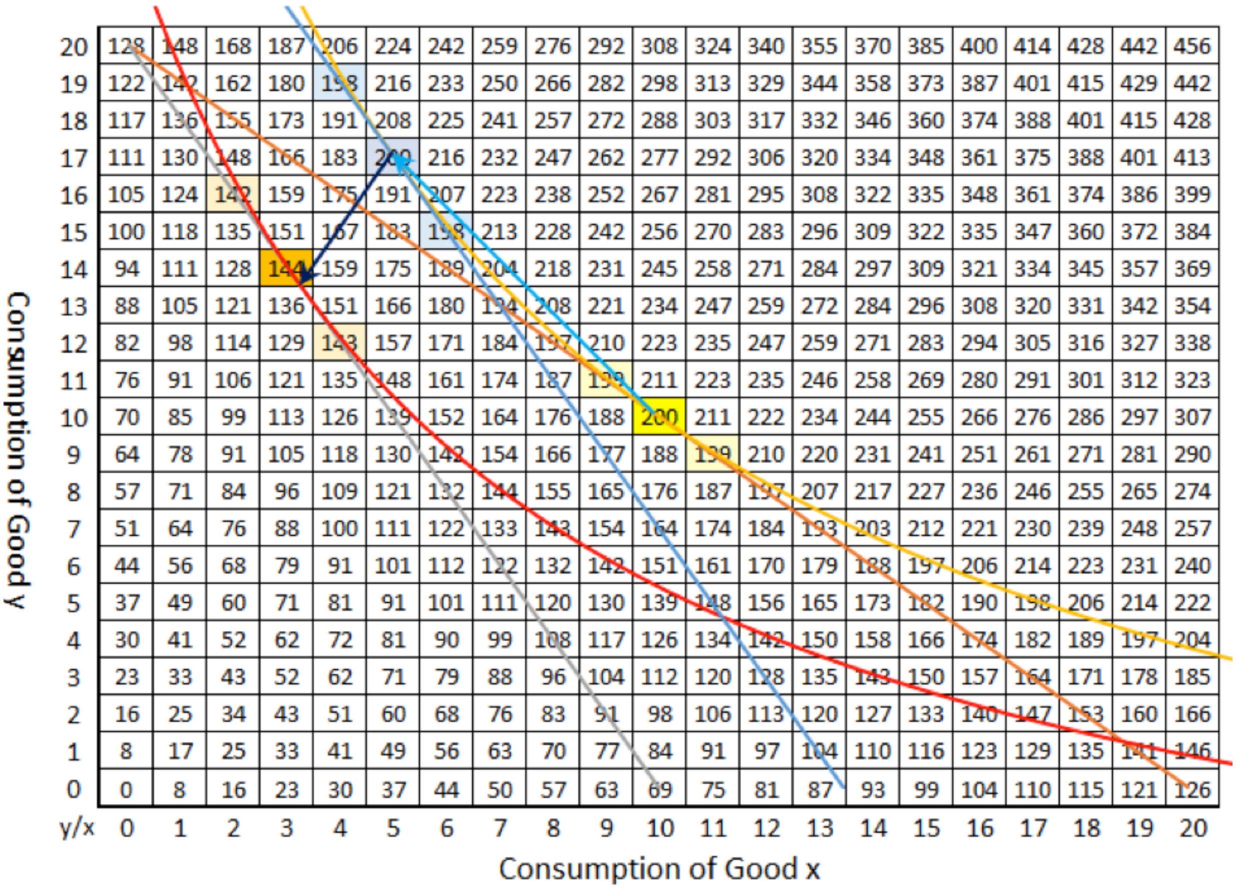
[8] The instructor may want to point out that budget constraints are shown as going through the center of a cell, from (0, 20) to (20, 0), for example. The Excel file described in the appendix lets the instructor discuss a table and add or remove various graphic elements that are overlaid on the table. This ability is especially useful for interactive lectures.

Table 5 – Overlay of Indifference Curves and Budget Constraints Showing the Substitution and Income Effects of an Increase in the Price of x when y is a Substitute for x



20	128	148	168	187	206	224	242	259	276	292	308	324	340	355	370	385	400	414	428	442	456
19	122	142	162	180	198	216	233	250	266	282	298	313	329	344	358	373	387	401	415	429	442
18	117	136	155	173	191	208	225	241	257	272	288	303	317	332	346	360	374	388	401	415	428
17	111	130	148	166	183	200	216	232	247	262	277	292	306	320	334	348	361	375	388	401	413
16	105	124	142	159	175	191	207	223	238	252	267	281	295	308	322	335	348	361	374	386	399
15	100	118	135	151	167	183	198	213	228	242	256	270	283	296	309	322	335	347	360	372	384
14	94	111	128	144	159	175	189	204	218	231	245	258	271	284	297	309	321	334	345	357	369
13	88	105	121	136	151	166	180	194	208	221	234	247	259	272	284	296	308	320	331	342	354
12	82	98	114	129	143	157	171	184	197	210	223	235	247	259	271	283	294	305	316	327	338
11	76	91	106	121	135	148	161	174	187	199	211	223	235	246	258	269	280	291	301	312	323
10	70	85	99	113	126	139	152	164	176	188	200	211	222	234	244	255	266	276	286	297	307
9	64	78	91	105	118	130	142	154	166	177	188	199	210	220	231	241	251	261	271	281	290
8	57	71	84	96	109	121	132	144	155	165	176	187	197	207	217	227	236	246	255	265	274
7	51	64	76	88	100	111	122	133	143	154	164	174	184	193	203	212	221	230	239	248	257
6	44	56	68	79	91	101	112	122	132	142	151	161	170	179	188	197	206	214	223	231	240
5	37	49	60	71	81	91	101	111	120	130	139	148	156	165	173	182	190	198	206	214	222
4	30	41	52	62	72	81	90	99	108	117	126	134	142	150	158	166	174	182	189	197	204
3	23	33	43	52	62	71	79	88	96	104	112	120	128	135	143	150	157	164	171	178	185
2	16	25	34	43	51	60	68	76	83	91	98	106	113	120	127	133	140	147	153	160	166
1	8	17	25	33	41	49	56	63	70	77	84	91	97	104	110	116	123	129	135	141	146
0	0	8	16	23	30	37	44	50	57	63	69	75	81	87	93	99	104	110	115	121	126
y/x	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

How does the consumer depicted in Table 6 respond to a doubling of the price of x? The utility maximizing bundle is to spend all income on y when the price of x is $2. The reason is straightforward, U(0, 20) = 177 > 176 = U(1, 18), and both bundles cost $20.

You can even work through what happens if instead the price of x had increased from $1 to $1.50 using Table 6. The bundle (0, 20) still costs $20 and the next bundle that is on the budget constraint is (2, 17), because $20 = $1.50·2 + $1·17. The bundle (2, 17) is preferable to (0, 20) because U(2, 17) = 181 > 177 = U(0, 20). Indeed, this individual will choose (2, 17) if P_x = $1.50 because the next bundle on the budget constraint has lower utility, U(2, 17) = 181 > 180 = U(4, 14).[9]

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[9] Erfle (2016) pp. 155-157, provides an expanded discussion of boundary solutions in the context of a wine-lover's decision to buy cabernet and merlot. Tables 6 and 7 are consistent with the following scenario: Suppose Mark wants to spend $200 on everyday cabernets and merlots. He likes both, but he likes cabernet a bit more than merlot. If both are $10 per bottle, Mark purchases 7 bottles of merlot (x) and 13 bottles of cabernet (y). If the price of merlot doubles to $20, Mark purchases exclusively cabernet, (0, 20). On the other hand, had the price of cabernet doubled and the price of merlot remained at $10,

Table 6 – An Indifference Map in which an Increase in the Price of x from \$1 to \$2 Leads to the Individual Purchasing Only y, Given I = \$20 and P_y = \$1

Consumption of Good y

20	177	191	203	215	226	237	246	256	265	273	281	289	296	303	310	317	323	329	335	341	347
19	170	184	196	208	219	229	239	248	257	265	273	281	288	295	302	309	315	321	327	333	339
18	163	176	189	201	211	222	231	241	249	258	266	273	280	287	294	301	307	313	319	325	330
17	156	169	181	193	204	214	224	233	241	250	258	265	272	279	286	293	299	305	311	316	322
16	148	162	174	185	196	206	216	225	233	242	249	257	264	271	278	284	290	296	302	308	313
15	141	154	166	178	188	198	208	217	225	233	241	249	256	263	269	276	282	288	293	299	305
14	133	146	158	170	180	190	199	208	217	225	233	240	247	254	260	267	273	279	285	290	296
13	125	138	150	161	172	182	191	200	208	216	224	231	238	245	252	258	264	270	275	281	286
12	117	130	142	153	163	173	182	191	199	207	215	222	229	236	242	249	255	260	266	272	277
11	109	121	133	144	155	164	174	182	190	198	206	213	220	227	233	239	245	251	257	262	267
10	100	113	125	136	146	155	164	173	181	189	196	204	210	217	223	230	235	241	247	252	257
9	91	104	116	126	137	146	155	164	172	179	187	194	201	207	214	220	225	231	237	242	247
8	82	95	106	117	127	137	146	154	162	170	177	184	191	197	203	209	215	221	226	232	237
7	73	86	97	108	118	127	136	144	152	160	167	174	180	187	193	199	205	210	216	221	226
6	64	76	87	98	108	117	126	134	142	149	156	163	170	176	182	188	194	199	205	210	215
5	54	66	77	88	97	107	115	123	131	139	146	152	159	165	171	177	183	188	193	199	204
4	44	56	67	77	87	96	104	113	120	128	135	141	148	154	160	166	171	177	182	187	192
3	34	45	56	66	76	85	93	101	109	116	123	130	136	142	148	154	159	165	170	175	180
2	23	34	45	55	65	73	82	90	97	104	111	118	124	130	136	142	147	152	157	162	167
1	12	23	34	44	53	62	70	78	85	92	99	105	112	118	123	129	134	139	144	149	154
0	0	11	22	32	41	49	57	65	72	79	86	93	99	105	110	116	121	126	131	136	141
y/x	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

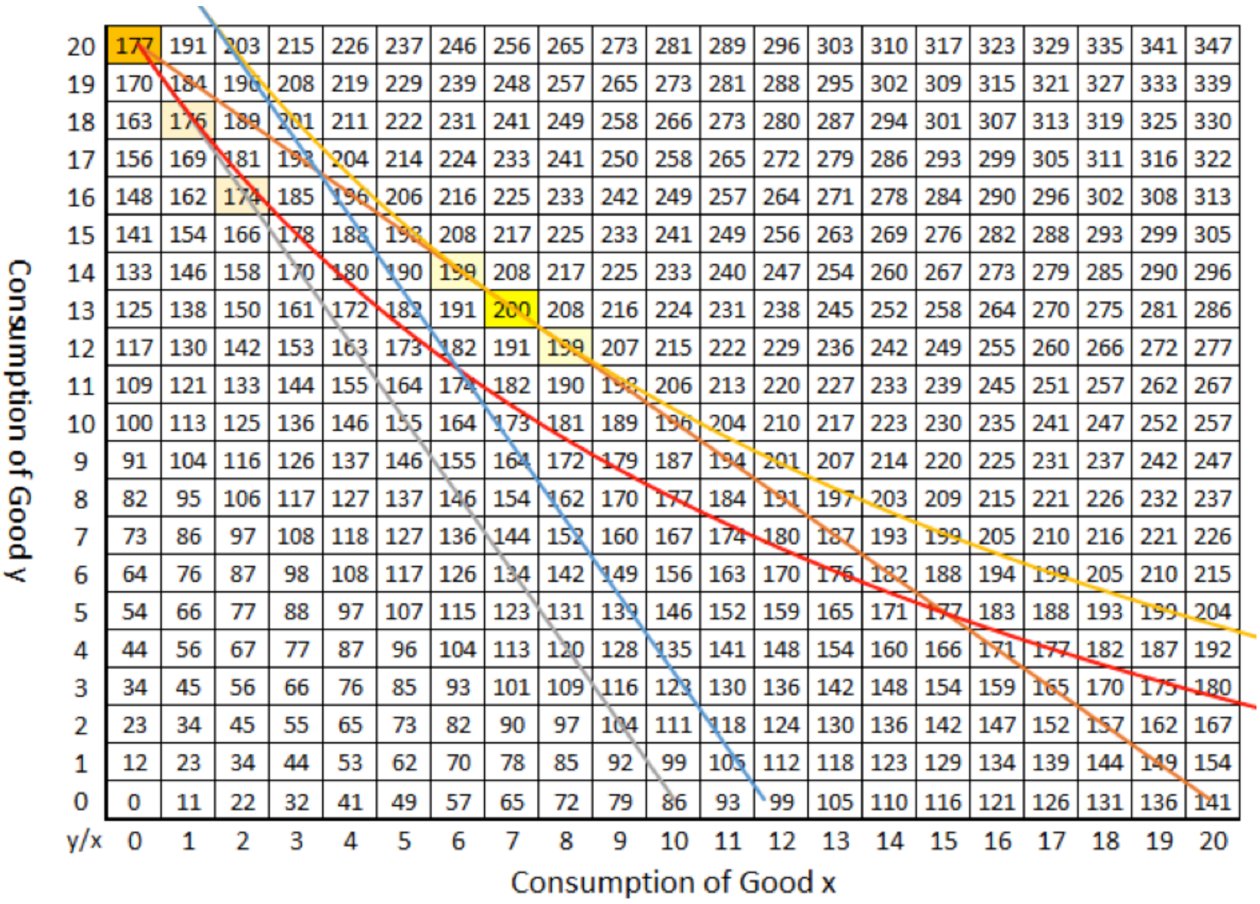
Consumption of Good x

Table 7 shows indifference curves and budget constraints associated with this situation.

Note the tangencies at (7, 13) and (0, 20). In this instance, the table does not show the substitution bundle [at (1, 21)] but the substitution budget constraint is provided. That way, instructors can work through the substitution and income effects to show that, once again, y is a substitute for x (because the cross-price substitution effect on y of +8 exceeds the cross-price income effect on y of -1).

Mark would be equally happy with 12 bottles of merlot and 4 bottles of cabernet or 14 bottles of merlot and 3 bottles of cabernet, because U(12, 4) = 148 = U(14, 3). However, had the price of cabernet tripled to \$30, Mark would purchase exclusively merlot, because U(17, 1) = 139 < 141 = U(20, 0). This behavior suggests that Mark considers cabernet and merlot to be imperfect substitutes.

Table 7 – Overlay of Indifference Curves and Budget Constraints for Table 6: An Increase



20	177	191	203	215	226	237	246	256	265	273	281	289	296	303	310	317	323	329	335	341	347
19	170	184	196	208	219	229	239	248	257	265	273	281	288	295	302	309	315	321	327	333	339
18	163	176	189	201	211	222	231	241	249	258	266	273	280	287	294	301	307	313	319	325	330
17	156	169	181	193	204	214	224	233	241	250	258	265	272	279	286	293	299	305	311	316	322
16	148	162	174	185	196	206	216	225	233	242	249	257	264	271	278	284	290	296	302	308	313
15	141	154	166	178	188	198	208	217	225	233	241	249	256	263	269	276	282	288	293	299	305
14	133	146	158	170	180	190	199	208	217	225	233	240	247	254	260	267	273	279	285	290	296
13	125	138	150	161	172	182	191	200	208	216	224	231	238	245	252	258	264	270	275	281	286
12	117	130	142	153	163	173	182	191	199	207	215	222	229	236	242	249	255	260	266	272	277
11	109	121	133	144	155	164	174	182	190	198	206	213	220	227	233	239	245	251	257	262	267
10	100	113	125	136	146	155	164	173	181	189	196	204	210	217	223	230	235	241	247	252	257
9	91	104	116	126	137	146	155	164	172	179	187	194	201	207	214	220	225	231	237	242	247
8	82	95	106	117	127	137	146	154	162	170	177	184	191	197	203	209	215	221	226	232	237
7	73	86	97	108	118	127	136	144	152	160	167	174	180	187	193	199	205	210	216	221	226
6	64	76	87	98	108	117	126	134	142	149	156	163	170	176	182	188	194	199	205	210	215
5	54	66	77	88	97	107	115	123	131	139	146	152	159	165	171	177	183	188	193	199	204
4	44	56	67	77	87	96	104	113	120	128	135	141	148	154	160	166	171	177	182	187	192
3	34	45	56	66	76	85	93	101	109	116	123	130	136	142	148	154	159	165	170	175	180
2	23	34	45	55	65	73	82	90	97	104	111	118	124	130	136	142	147	152	157	162	167
1	12	23	34	44	53	62	70	78	85	92	99	105	112	118	123	129	134	139	144	149	154
0	0	11	22	32	41	49	57	65	72	79	86	93	99	105	110	116	121	126	131	136	141
y/x	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

C. An Example Where x and y Are Complements

The final two tables provide an example of the reverse situation, where x and y are complementary goods. The same basic price adjustment is under consideration as in the last four tables. Suppose the price of x increases from \$1 to \$2. How would the consumer respond, given P_y = \$1 and I = \$20?

Given a price of x of \$1, the consumer maximizes utility by choosing 8 units of x and 12 units of y, bundle (8, 12), in Table 8. If the price of x increases to \$2, the consumer maximizes utility by choosing 5 units of x and 10 units of y, bundle (5, 10). Note that, in the initial situation, 40 percent of income is spent on x. Once the price of x increases, that percentage increases to 50 percent. The reverse held true in Tables 4 through 7. When goods are substitutes, a utility-maximizing consumer increases consumption of the good that is becoming less expensive on a relative basis. When goods are complements, a utility maximizing consumer decreases consumption of both goods.

Table 8 – An Indifference Map Where x and y Are Complementary Goods

Consumption of Good y

20		86	120	155	190	209	222	232	241	249	255	261	267	272	276	280	284	288	292	295	298
19		84	118	152	187	205	218	228	237	244	251	257	262	267	271	276	280	283	287	290	293
18		83	116	150	183	201	214	224	233	240	246	252	257	262	266	270	274	278	281	285	288
17		81	114	147	179	197	210	220	228	235	241	247	252	257	261	265	269	272	276	279	282
16		79	111	143	176	193	205	215	223	230	236	242	247	251	255	259	263	267	270	273	276
15		77	109	140	172	188	200	210	218	225	231	236	241	245	250	253	257	260	264	267	270
14		75	106	137	167	184	195	205	212	219	225	230	235	239	243	247	250	254	257	260	263
13		73	103	133	163	179	190	199	206	213	219	224	228	232	236	240	243	247	250	253	255
12		71	100	129	157	173	184	193	200	206	212	217	221	225	229	233	236	239	242	245	247
11		69	96	124	152	167	178	186	193	199	204	209	213	217	221	224	228	231	234	236	239
10		66	92	119	146	160	170	179	185	191	196	201	205	209	212	215	219	221	224	227	229
9		63	88	114	139	153	162	170	177	182	187	191	195	199	202	205	208	211	214	216	218
8		59	83	107	131	144	153	161	167	172	176	180	184	188	191	194	196	199	202	204	206
7		55	77	99	122	134	142	149	155	159	164	168	171	174	177	180	182	185	187	189	191
6		50	70	90	110	121	129	135	140	144	148	151	155	157	160	163	165	167	169	171	173
5		42	59	76	94	103	109	114	119	123	126	129	131	134	136	138	140	142	144	145	147
4		27	38	49	60	66	70	73	76	79	81	83	84	86	87	89	90	91	92	93	94
3		18	28	32	39	43	47	50	53	56	58	61	63	65	68	70	72	74	76	78	80
2		9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60	63	66
1		1	3	5	7	9	11	13	15	17	19	21	23	25	27	29	31	33	35	37	39
0																					
y/x	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

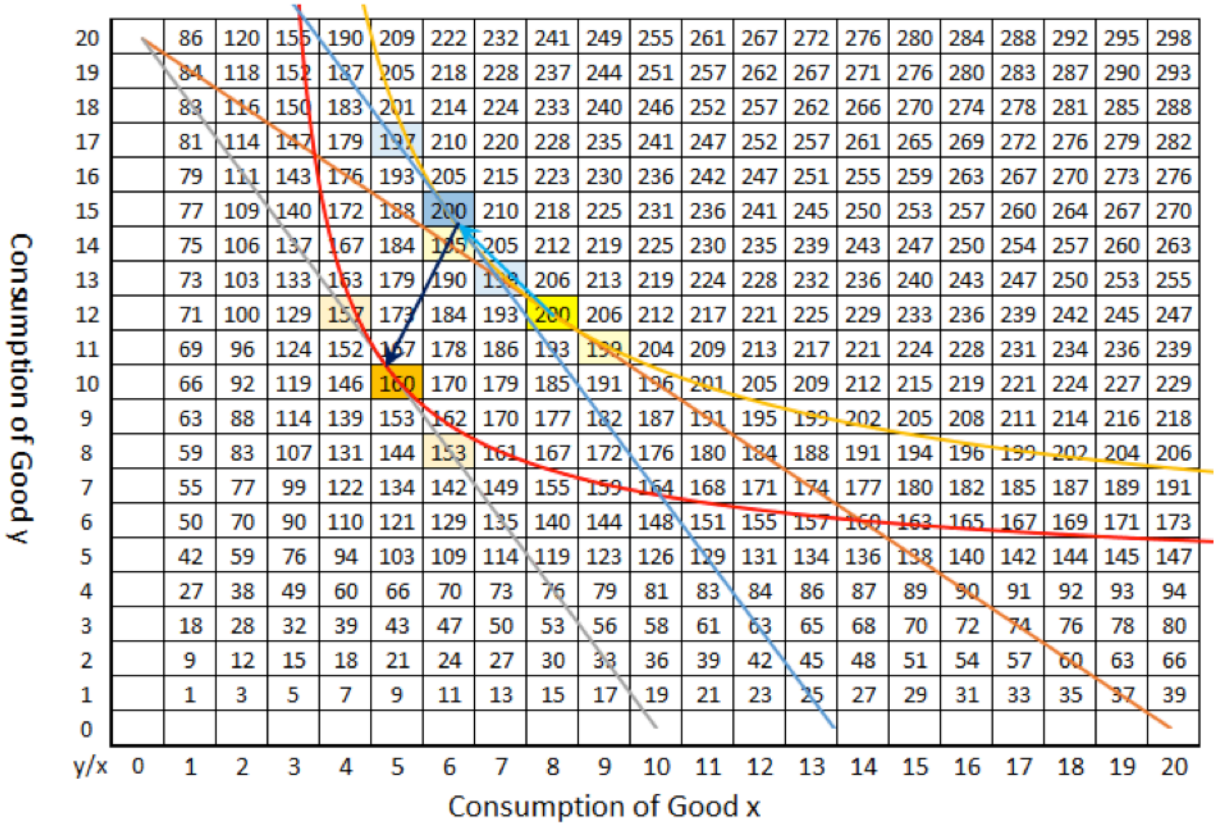
Consumption of Good x

As above, we can reinforce the analysis by examining the substitution bundle. The substitution bundle in this instance is (6, 15) because $MU_x/P_x = 10/\$2 = 5/\$1 = MU_y/P_y$. Once again, x and y are normal goods because the income effect of the price increase is negative for both goods. By using the cross-price view, we can also see that x and y are complements. The cross-price substitution effect is +3 (from 12 to 15) and the cross-price income effect is -5 (from 15 to 10), so the net effect of an increase in price of x on the demand for y is negative (-2 = +3 – 5). This is a numerical example that exactly highlights the textbook definition of y being a complement to x. Geometrically, we see that y is a complement to x if the cross-price income effect dominates the cross-price substitution effect.

Instructors could then pose the required compensation question. In this instance, the answer is $7 because (6, 15) costs $27 given $P_x = \$2$. Because the individual already has $20, $7 extra is required to maintain the initial level of utility after this price increase.

One might follow this up by showing a graphical overlay of indifference curves and budget constraints highlighting the tangency condition that is equivalent to the equal-bang-for-the-buck rule. Table 9 shows the completed graph including the substitution and income effects. Note in particular that the income effect on y dominates the substitution effect on y of the increase in the price of x.

Table 9 – Overlay of Indifference Curves and Budget Constraints Showing the Substitution and Income Effects of an Increase in the Price of x when y is a Complement of x



Consumption of Good y

20		86	120	155	190	209	222	232	241	249	255	261	267	272	276	280	284	288	292	295	298
19		84	118	152	187	205	218	228	237	244	251	257	262	267	271	276	280	283	287	290	293
18		83	116	150	183	201	214	224	233	240	246	252	257	262	266	270	274	278	281	285	288
17		81	114	147	179	197	210	220	228	235	241	247	252	257	261	265	269	272	276	279	282
16		79	111	143	176	193	205	215	223	230	236	242	247	251	255	259	263	267	270	273	276
15		77	109	140	172	188	200	210	218	225	231	236	241	245	250	253	257	260	264	267	270
14		75	106	137	167	184	195	205	212	219	225	230	235	239	243	247	250	254	257	260	263
13		73	103	133	163	179	190	198	206	213	219	224	228	232	236	240	243	247	250	253	255
12		71	100	129	157	173	184	193	200	206	212	217	221	225	229	233	236	239	242	245	247
11		69	96	124	152	167	178	186	193	199	204	209	213	217	221	224	228	231	234	236	239
10		66	92	119	146	160	170	179	185	191	196	201	205	209	212	215	219	221	224	227	229
9		63	88	114	139	153	162	170	177	182	187	191	195	199	202	205	208	211	214	216	218
8		59	83	107	131	144	153	161	167	172	176	180	184	188	191	194	196	199	202	204	206
7		55	77	99	122	134	142	149	155	159	164	168	171	174	177	180	182	185	187	189	191
6		50	70	90	110	121	129	135	140	144	148	151	155	157	160	163	165	167	169	171	173
5		42	59	76	94	103	109	114	119	123	126	129	131	134	136	138	140	142	144	145	147
4		27	38	49	60	66	70	73	76	79	81	83	84	86	87	89	90	91	92	93	94
3		18	28	32	39	43	47	50	53	56	58	61	63	65	68	70	72	74	76	78	80
2		9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60	63	66
1		1	3	5	7	9	11	13	15	17	19	21	23	25	27	29	31	33	35	37	39
0																					
y/x	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

Consumption of Good x

6. Conclusion

This paper suggests extensions for classroom analysis using the framework suggested by Holmgren (2017). These extensions allow instructors to analyze the equal-marginal-utility-per-dollar-spent consumer choice rule discussed in introductory textbooks using discrete utility tables. One can also examine the ordinal nature of utility using these tables. Additionally, the rate that an individual is willing to give up y to get one more x, the marginal rate of substitution, is readily examined using discrete choice utility tables.

Discrete choice utility tables allow instructors to examine the relation between two goods. Introductory texts describe three types of relationships between two goods (such as x and y) from a consumer's perspective. Two goods may be substitutes for one another, they may be complements to one another, or they may be independent of one another. Each definition can be viewed in the choices that an individual consumer makes when a price change occurs. These choices differ, for different kinds of preferences. This tabular approach reinforces the written definitions provided in introductory texts and allows students to see these concepts from multiple perspectives.

Finally, graphical overlays are provided that may be used for classroom discussion in order to tie the discrete choices involved in the tables to their continuous counterparts. These graphic overlays are most readily employed in a classroom setting using the Excel file for this paper.

Although each of the strategies discussed in this paper are appropriate for an introductory microeconomics classroom, they could also be employed to kick-start a more in-depth discussion of these same topics in intermediate microeconomics classes. As noted in the introduction, they could also be used in elective classes that do not require intermediate level economics.

References

Erfle, S.E. (2016). *Intermediate Microeconomics: An Interactive Approach*. Saint Paul, MN: Textbook Media Press.

Holmgren, M. (2017). From continuous to discrete: An alternative approach to teaching consumer choice. *Journal of Economics Teaching, 2*(1), 1-13.

Appendix A – Working with the Excel file that Created the Tables for this Paper

Note: To simplify discussion, components of and commands in the Excel file are noted in Bold below.

This paper uses a series of discrete choice utility tables, each of which was created using the Macro-Enabled Excel file accessible through this link. Excel Screenshot 1 shows parts of the **Base** sheet, the top six rows of which are the dashboard that controls the file. Part A shows elements that allow the user to alter the underlying preferences and graphic elements representing those preferences. The table shown at the bottom of Part A is Table 3 with graphical overlay showing the income and substitution effect of a price increase discussed in Section 4. This is, of course, the equally weighted Cobb-Douglas utility function, U(x, y) = x·y.

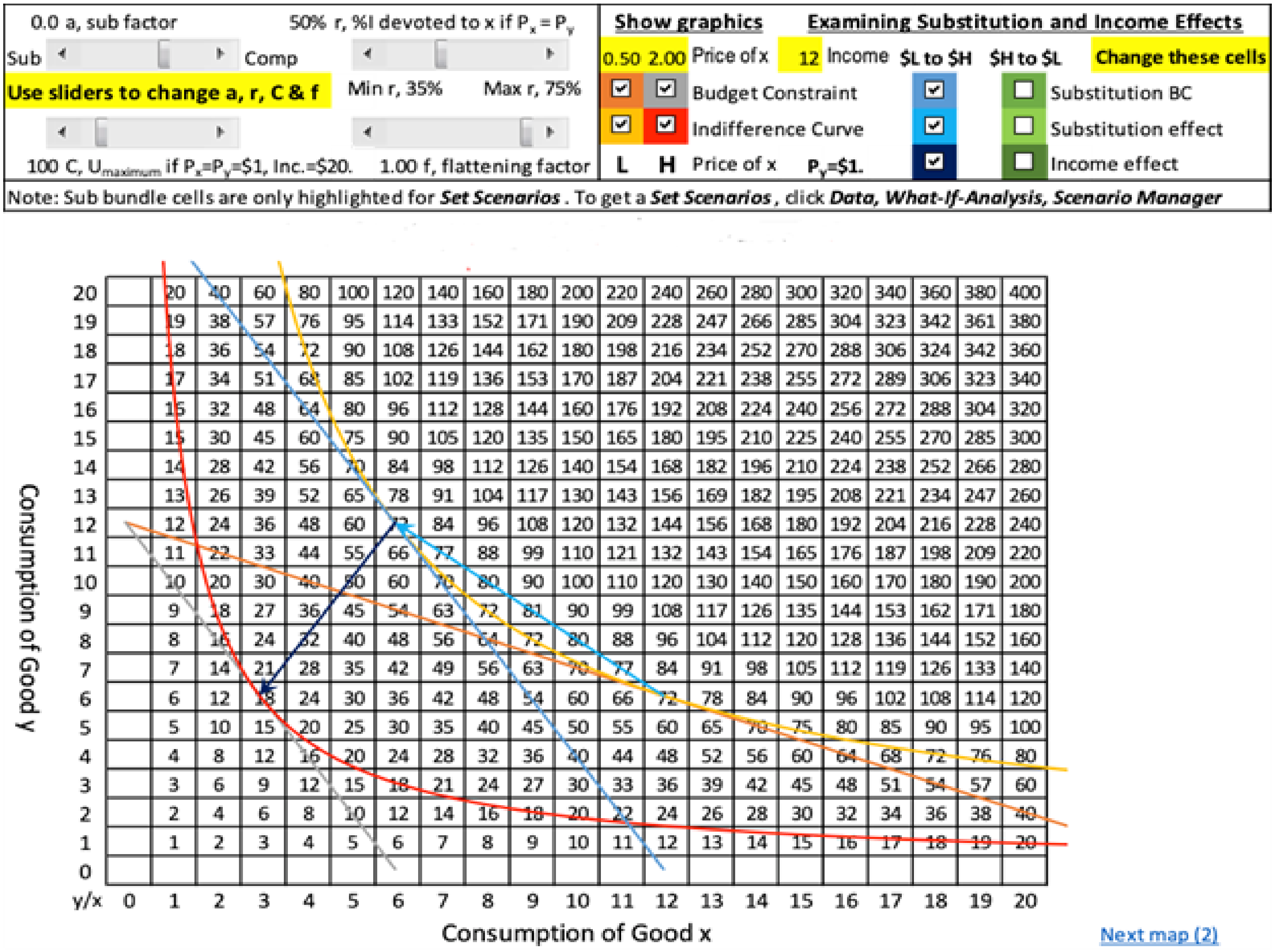
The rest of the tables are versions of a shifted Cobb-Douglas utility function. The specific functional form need not concern the reader. The file works for non-equally weighted Cobb-Douglas utility functions because the non-integer solutions can be made to appear to be integers by requesting that no decimal values be provided. Various versions of each table are provided with hyperlinks as noted in Part B. These hyperlinks within the **Base** sheet allow the instructor to build out their analysis for students using overhead projection. Part C simply provides a close-up view of part of the table so they can talk about individual marginal utilities as discussed in Section 3.b. The instructor can alter highlighted x and y values and can determine the utility level of that bundle and the marginal utilities that result.

The tables used in the paper can be obtained using the **Scenarios** function from the **Base** sheet. To request a table, the instructor should click **Data**, **What-If Analysis**, **Scenario Manager**, and then choose the scenario they wish to see. The **Utility Tables** sheet also provides hard-copy versions of some sheets because extensive highlighting was done (see Tables 1 and 2 in Section 3.a, for example).

When creating a new version of the utility table, the instructor can use the graphic overlays to find versions where the substitution arrow starts and stops close to the center of a cell. If utility values are not identical, they will be very close. Modestly adjusting the flattening factor (shown in **I5**, slider in **H4:M4**) will allow the instructor to nudge the two to the same value. Similarly, the substitutability coefficient (shown in **A1,** slider in **A1:E1**) can be moved 0.1 per click using the arrow endpoints. Such moves will change numbers but have minimal impact on the graphical solution.

The **Price & Income Scenarios** sheet is discussed in Section 4.c. The **Figure 1** sheet allows the instructor to project an interactive version of Figure 1 for students. Rather than set V to $10 \cdot (x \cdot y)^{0.5}$ as shown in the paper, a more general version is provided. This version sets $V = a \cdot (x \cdot y)^{b}$ and sliders allow the instructor to vary a and b and a clickbox allows them to show or hide U and V utility values. The **Affordability** sheet rounds out the file. It provides the instructor the ability to discuss affordable bundles. Income and the price of x are manually entered into cells **A1** and **A2**. For example, if the instructor puts in 0.75 in **A2**, they can see that the budget constraint is binding each time the individual consumes three fewer units of y they can get 4 more units of x. It is important to note that the values in this table do not refer to utility; they refer to cost. This is the only place in the file where this is the case. Elsewhere, budget elements are shown via graphic overlay or by highlighted cells with utility values in each cell.

Excel Screenshot 1. Part A – Dashboard Showing Sliders to Change Preferences, Highlighted Cells Control Price of x and Income, and Click-boxes Control Graphic Overlay Elements: Table 3, P_x ↑Case Shown Below



0.0 a, sub factor

50% r, %I devoted to x if $P_x = P_y$

Sub ◂ ▸ Comp ◂ ▸

Use sliders to change a, r, C & f

Min r, 35% Max r, 75%

◂ ▸ ◂ ▸

100 C, $U_{maximum}$ if P_x=P_y=$1, Inc.=$20.

1.00 f, flattening factor

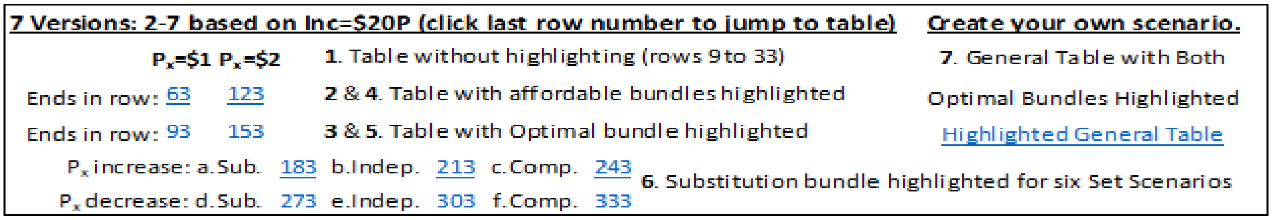
Show graphics **Examining Substitution and Income Effects**

0.50	2.00	Price of x	12	Income	**$L to $H**	**$H to $L**	**Change these cells**
☑	☑	Budget Constraint			☑	☐	Substitution BC
☑	☑	Indifference Curve			☑	☐	Substitution effect
L	**H**	Price of x		P_y=$1.	☑	☐	Income effect

Note: Sub bundle cells are only highlighted for ***Set Scenarios***. To get a ***Set Scenarios***, click ***Data, What-If-Analysis, Scenario Manager***

20		20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400
19		19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361	380
18		18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342	360
17		17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323	340
16		16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320
15		15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300
14		14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266	280
13		13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247	260
12		12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240
11		11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209	220
10		10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200
9		9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180
8		8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160
7		7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140
6		6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120
5		5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
4		4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80
3		3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60
2		2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40
1		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0																					
y/x	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

Part B – Hyperlinks to Set Scenarios for Classroom Use Showing Versions of Tables 4-9



7 Versions: 2-7 based on Inc=$20P (click last row number to jump to table)

$P_x=\$1$ $P_x=\$2$

Ends in row: 63 123

Ends in row: 93 153

P_x increase: a.Sub. 183 b.Indep. 213 c.Comp. 243

P_x decrease: d.Sub. 273 e.Indep. 303 f.Comp. 333

1. Table without highlighting (rows 9 to 33)

2 & 4. Table with affordable bundles highlighted

3 & 5. Table with Optimal bundle highlighted

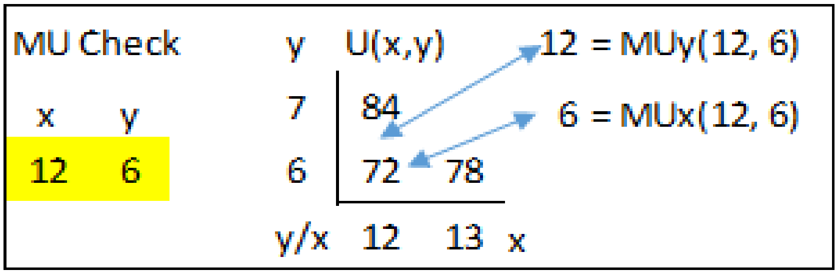
6. Substitution bundle highlighted for six Set Scenarios

Create your own scenario.

7. General Table with Both Optimal Bundles Highlighted

Highlighted General Table

Part C – Marginal Utility Check Tool: Change x and y in Yellow Cells to See How MU_x and MU_y Vary



MU Check

x	y
12	6

y	U(x,y)	
7	84	
6	72	78
y/x	12	13 x

12 = MUy(12, 6)

6 = MUx(12, 6)